Math 21(2010)

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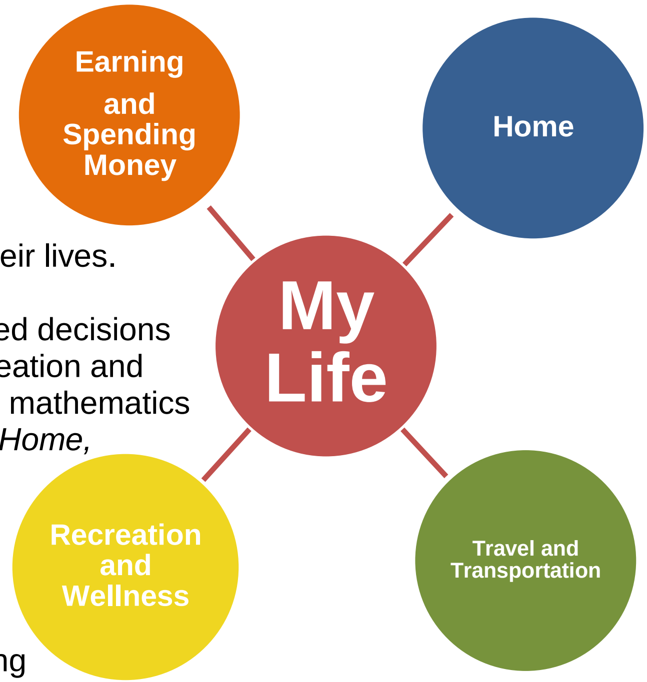
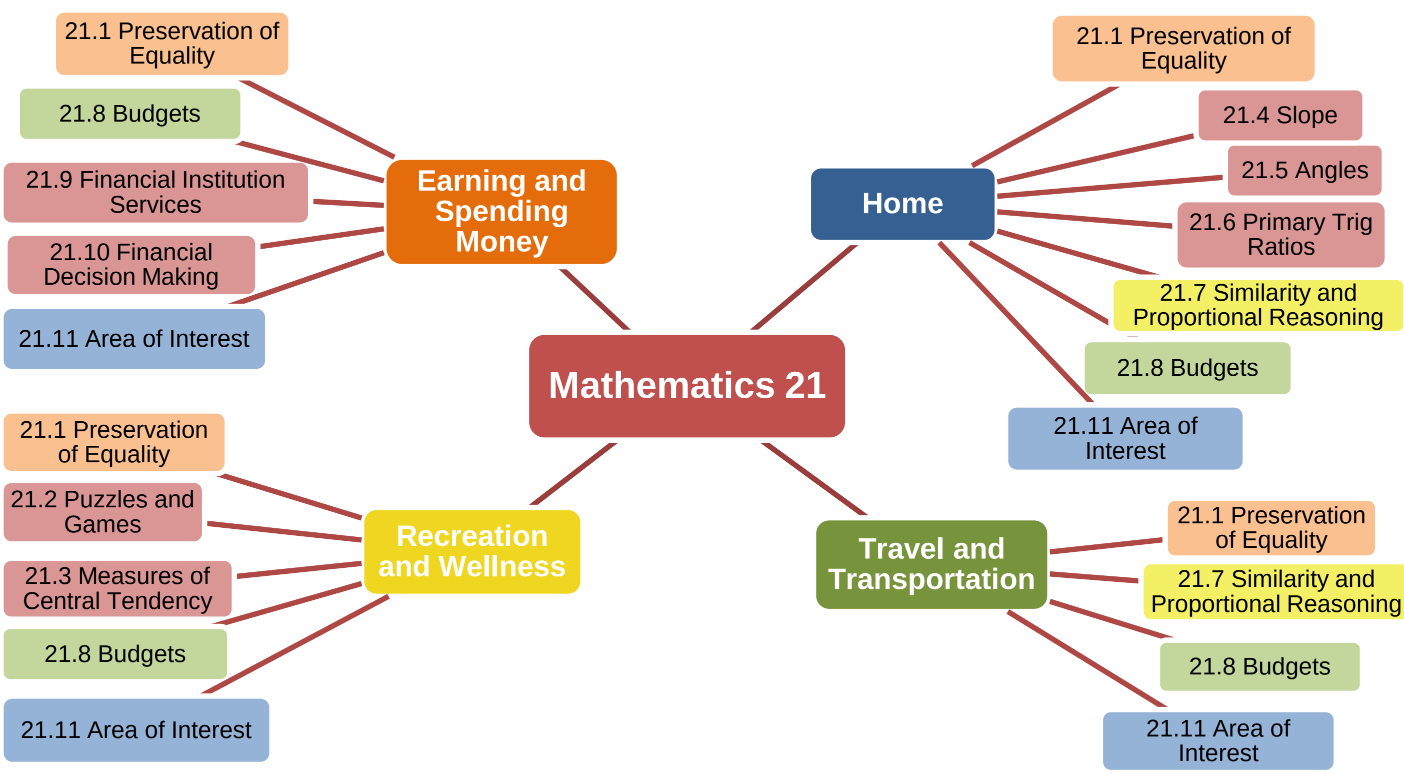
Introduction

Prerequisite: Mathematics 11, Foundations and PreCalculus 10, and/or Workplace and Apprenticeship 10



This course is designed for theme-based instruction, which should enable students to broaden their understanding of mathematics as it is applied in important areas of day-to-day living. There is a need for learning to be meaningful in order to be transferable. Learning mathematics should provide students an opportunity to explore mathematics in their lives.

In this course, emphasis is placed on making informed decisions about finances, home design and maintenance, recreation and personal wellness, and travel and transportation. All mathematics relate to the themes: *Earning and Spending Money, Home, Recreation and Wellness,* and *Travel and Transportation.* Students can draw on their own or others experiences in the workforce to develop and extend their knowledge about earning and spending money. They will also apply mathematics for the purpose of designing, building, and maintaining a home and yard. Students will apply reasoning and problem solving skills to make predictions and decisions in recreational and wellness activities. As well, they will investigate and solve problems related to planning a trip.



Adaptive Dimension

In order to meet the variety of students' needs, flexibility is required within the school program to enable schools and teachers to adapt instructional materials, methods, and the environment to provide the most appropriate educational opportunities for students.

The Adaptive Dimension is used to:

- help students achieve curriculum outcomes
- maximize student learning and independence
- lessen discrepancies between achievement and ability
- promote a positive self-image and feeling of belonging
- promote a willingness to become involved in learning
- provide opportunities for all students to be engaged in learning.

These purposes address a primary function of the school, that of helping students to maximize their potentials as independent learners (Ministry of Education, *Core Curriculum Components and Initiatives*, December 17, 2007).

The intent of the Adaptive Dimension applies to all programs and courses of instruction. The key variables of instruction are differentiated – the content (what students will learn), the learning processes (how students will interact with the content), and the learning products (how students will demonstrate learning and mastery of content), and the instructional setting or environment.

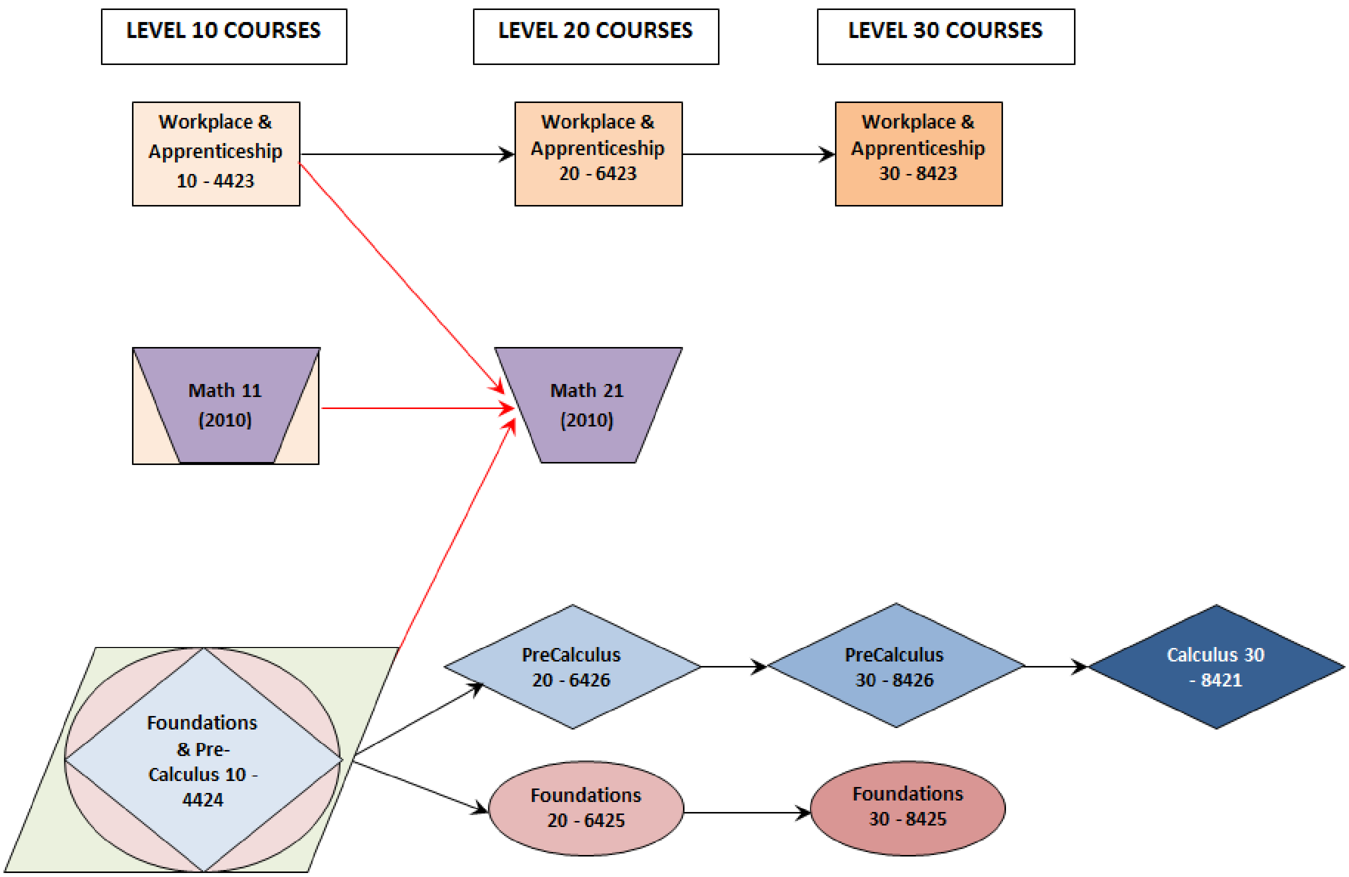
Some students may not be able to complete a Foundations of Mathematics 20, Pre-calculus 20 and/or Workplace and Apprenticeship 20 course even though adaptations to curriculum materials and topics, instruction, and environment have been made. This may require the development of a modified (Mathematics 21) course to meet student needs to which the Adaptive Dimension may be applied.

Modified Courses

Careful assessment and diagnosis are necessary to understand the language and learning abilities and needs of modified students and to inform instruction. It is important to recognize that the student's academic strengths and weaknesses must be determined through formal and informal means prior to a placement being recommended. If it has been determined through assessment, observation, and collaborative team meetings that a student's needs are best met through placement in a Mathematics 21 course, then all those involved in this decision must carefully consider the implications of such a placement. Everyone including parents/caregivers, students, teachers, and administrators must review the *Policy and Procedures for Locally Modified Courses of Study* with particular attention given to the rationale and implications as outlined. Learning disabilities and/or behaviour disorders should not be the sole criteria for placement in a modified course. Nor, is it appropriate to place students in a modified course because the usual language of instruction is their second language or a non-standard dialect.

Credit Information

Mathematics 21 is not a like credit to the other level 20 courses, Workplace & Apprenticeship 20, PreCalculus 20, and Foundations 20, therefore a student can gain credit for all four level 20 courses.



Pre-requisite

Modified pathway

Credit Information:

1. Same colour, vertical difference = like credits
2. Same colour family = general pathway
3. Graduation requirements: 1 credit from the Level 10 column and 1 credit from the level 20 column
   - Level 20 credits are suppressed until an appropriate level 10 credit is granted
   - Level 30 credits are suppressed until appropriate level 10 and 20 credits are granted

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2011) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website for various policy and foundation documents.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

Defined below are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes will therefore, also promote student achievement with respect to the K-12 goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships

- modeling and representing (including concrete, oral, physical, pictorial, and other symbolic representations)
- conjecturing and asking “what if” (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students’ number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon patterns that are discovered should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)
- build self-confidence related to mathematical insights and abilities

- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning which values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this course is often viewed in schools and schooling through a Western or European lens, but there are many different lenses, such as those of many First Nations and Métis peoples, through which mathematics can be viewed and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger students will become in their number sense, spatial sense, and logical thinking.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Critical Characteristics of Mathematics Education

The following sections highlight some of the different facets for teachers to consider in the process of changing from “covering” to supporting students in “discovering” mathematical concepts. These facets include:

- the seven mathematical processes
- the difference between covering and discovering mathematics
- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Mathematical Processes

This Mathematics 21 curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing, along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase students’ willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. When estimating, students need to know what strategy to use, when to use it, and how to use it.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you ...?", "Can you ...?", or "What if ...?", the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students are given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning for mathematical instruction and learning.

The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols and quantities.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies, processes, and rules, as well as the students' current and intuitive understandings of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that are already known or that make sense to students.

First Nations and Métis Learners and Mathematics

Teachers must recognize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understanding. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught in relation to their schema, cultural and environmental context, or real life experiences.

A first step in the actualization of mathematics from First Nations and Métis perspectives is empowering teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional Western European ways of teaching mathematics also are culturally biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understanding and mathematical self-confidence and ability through a more holistic and constructivist approach to teaching. Teachers need to pay close attention to those factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

Teachers must recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding, and as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas among cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. In addition, they also allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. As well, ethnomathematics demonstrates the relationship between mathematics and cultural anthropology.

Individually and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthens the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in students' understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy.

Critiquing statements is an effective way to assess students individually, as a small group, or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important, in learning mathematics, that students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, teachers must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you represent what you've done here using mathematical symbols?) or by having students compare their representations with others in the class. In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help to consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of drill will vary among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information to teacher’s planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students’ deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflecting
- exploring of patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are involved and engaged directly in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge.

Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry, and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning should also be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.

- are open-ended, whether in answer or approach, as there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but encourage students to make connections and generalizations.
- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p. 3)

Organization of Outcomes and Themes

For ease of reference, the outcomes in this document are numbered using the following system M21.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that M21.1 need not be taught before M21.2.

There are four themes in this course, *Earning and Saving Money, Home, Recreation and Wellness,* and *Travel and Transportation*. The ordering and grouping of the themes in Mathematics 21 is at the discretion of the teacher. Teachers are encouraged to design learning activities that integrate outcomes from throughout the course so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics.

Introduction of Outcomes and Indicators

The outcomes in the Mathematics 21 course are based upon the students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade. The mathematical knowledge and skills acquired through this course will be useful to students in many applications throughout their lives in both work and non-work settings.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can and should be created by teachers to meet the needs and circumstances of their students and communities.

The Mathematics 21 outcomes have included some outcomes and indicators from the Foundations pathway, and Workplace and Apprenticeship Mathematics pathway outcomes. The outcomes and indicators written in **green font** are from the current Foundations or Workplace and Apprenticeship Mathematics outcomes; the additional outcomes and indicators written in **black font** are unique to the Mathematics 21 course. The intent of the different colours of font is to assist teachers who teach combined classes.

Outcomes and Indicators

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA10.1 and WA20.1] **M21.1 Extend and apply understanding of the preservation of equality by solving problems that involve the manipulation and application of formulae within home, money, recreation, and travel themes.** **[C, CN, ME, R, V]**	a. Verify whether given forms of the same formula are equivalent and justify the conclusion. b. Describe, using examples, how a given formula is used in a home, money, recreation, and travel context. c. Create, solve, and verify the reasonableness of solutions to questions that involve the use of a formula. d. Analyze solutions to questions that involve formulae to verify the preservation of equality, correct if necessary, and explain the reasoning. e. Solve, with or without the use of technology, questions that involve the application of a formula that: • does not require manipulation • does require manipulation. ***Sample Formulae:*** *volume and capacity, surface area, slope and rate of change, primary trigonometric ratios, finance charges, and income.*

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[FM20.2 and WA20.2]	
M21.2 Demonstrate understanding of numerical reasoning and problem solving strategies by analyzing puzzles and games. [C, CN, PS, R]	a. Make conjectures by observing patterns and identifying properties, and justify the reasoning. b. Observe and analyze errors in solutions to puzzles or in strategies for winning games to identify and correct errors, if necessary, and explain the reasoning. c. Solve questions that involve numerical reasoning. d. Create a variation of a puzzle or a game, and describe a strategy for solving the puzzle or winning the game. ***Sample Games****: Cribbage, Magic Square, Yahtzee, Sudokos, Kakuro, Kaponk, Guesstamations, and Qwirkle.*

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA30.9]	
M21.3 Extend and apply understanding of measures of central tendency to analyze	a. Examine the distribution of a set of data, using smallest and largest value, frequency, value in the

data.	middle and patterns.
[C, CN, PS, R]	b. Explain, using examples, the advantages and disadvantages of each measure of central tendency.
	c. Explain the appropriate use of measures of central tendency, including mean, mode, and median.
	d. Calculate and interpret measures of central tendency, including mean, mode and median, to solve problems (e.g. batting averages, target heart rate, average temperature).
	e. Compare two or more sets of data, using measures of central tendency.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA20.9]	
M21.4 Demonstrate understanding of slope. **[CN, PS, R, T, V]**	a. Research and present contexts that involve slope including the mathematics involved (e.g., ramps, roofs, road grade, skateboard parks, ski hills, treadmill). b. Critique the statement, “It requires less effort to independently use a wheelchair to climb a ramp of a certain height that has a slope of 1:12 rather than a slope of 1:18." c. Justify, using examples and illustrations slope as rise over run. d. Analyze slopes of objects, such as ramps or roofs,

to determine if the slope is constant and explain the reasoning.

e. Analyze, generalize, and explain, using illustrations, the relationship between slope and angle of elevation (e.g., for a ramp (or pitch of a roof, grade of a road, slope in pipes for plumbing, azimuth in the sky) that has a slope of 7:100, the angle of elevation is approximately 4 degrees).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA10.9]	
M21.5 Demonstrate understanding of angles created by parallel, perpendicular, and transversal lines and solve problems within the home theme. **[C, ME, PS, T, V]**	a. Observe and sort pairs of lines as perpendicular, parallel, or neither, and justify the reasoning (e.g., tiling, ceiling tiles, flooring, framing, cutting a window frame). b. Generalize, develop, explain, and apply relationships between pairs of angles formed by parallel lines and a transversal, including: • corresponding angles • vertically opposite angles • alternate interior angles • alternate exterior angles • interior angles on the same side of the transversal • exterior angles on the same side of the transversal. c. Provide concrete and pictorial examples that show that there are no angle relationships (excluding vertically opposite angles) when two lines that are not parallel are crossed by a transversal.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA10.8]	
M21.6 Demonstrate understanding of primary trigonometric ratios (sine, cosine, and tangent). **[CN, PS, R, T, V]**	a. Describe the properties of a triangle. b. Determine the missing angle in a triangle. c. Observe a set of similar right triangles and analyze and draw conclusions about the ratios of the lengths, with respect to one acute angle of the: • side opposite to the side adjacent • side opposite to the hypotenuse • side adjacent to the hypotenuse. d. Apply formulae for the primary trigonometric ratios (cosine, tangent, and sine). e. Describe, using examples, how a trigonometric formula is used in the home context. f. Analyze solutions to questions that involve primary trigonometric ratios to determine if they are reasonable and explain the reasoning.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	**Indicators**
[FM20.3]	
M21.7 Demonstrate and extend understanding of similarity and proportional reasoning related to scale factors, scale drawing, scale models, surface area, and volume.	a. Explain how ratios and proportionality are related to similarity of shapes.
[C, CN, PS, R, V]	b. Explain how scale factor is related to similarity, ratios, and proportionality.
	c. Draw enlargements and reductions to scale.
	d. Determine the scale factor from scale drawings.
	e. Describe the relationship between scale factors, scale drawings, and maps.
	f. Interpret directions and analyze locations using scale factors and scale drawings of maps.
	g. Determine distances represented on maps (e.g. provincial road map, local street map, Web-based maps), using given scales.
	h. Explain the effect of a change in scale factor on the area of a 2-D shape or the surface area or volume of a 3-D object.
	i. Draw a scale diagram of a 2-D shape to a specified scale factor (enlargement or reduction) and examine and describe the strategies used.
	j. Draw a scale drawing of a familiar setting (e.g., classroom, bedroom, playground).
	k. Manipulate concrete 3-D objects to identify, describe, and sketch top, front, and side views.
	l. Analyze a set of views of 3-D objects to determine if they represent a given object and explain the

reasoning.

m. Construct models of 3-D objects, given the top, bottom, and side views.

n. Design and construct a 3-D scale model of an object or space (e.g., bedroom, hockey rink, ice fishing shack, dog house).

o. Describe the relationship between the area of the base and the volume of a 3-D object.

p. Solve situational questions involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figure (e.g., refrigerator or freezer, soil, cement, gravel, grain bins, sheds).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA20.6]	
M21.8 Demonstrate understanding of budgets. **[CN, PS, R, T, V]**	a. Identify fixed and variable expenses that could be included in a personal budget. b. Explain considerations that must be made when developing a budget (e.g., prioritizing, recurring and unexpected expenses). c. Research the costs of expenses (e.g., bus pass, rent, phone, electricity, power, groceries) to create and justify a personal budget. d. Analyze and modify a budget to achieve a set of personal goals.

e. Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

f. Explain the advantages and challenges of creating personal budgets.

g. Record and monitor purchases to determine personal expenditures.

h. Investigate, plan, design, and prepare a budget to solve home, recreation, or travel problems using appropriate technologies (e.g., design or decorating websites, design or drawing software, spreadsheet).

i. Create a monthly transportation budget that involves the fixed costs (e.g., licence fee, insurance) and variable costs (e.g., maintenance, fuel) of owning and operating a vehicle.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA20.7 and WA20.8]	
M21.9 Demonstrate understanding of financial institution services. **[CN, PS, R, T, V]**	a. Research various types of banking services available from various financial institutions, such as online services, different types of accounts, telephone banking, mobile banking, ATM banking, or cheques. b. Consider the services that banking institutes and financial advisors offer to assist in personal budgeting. c. Analyze the type of account that best meets the criteria for the provided examples and personal situations. d. Research and explain various charges acquired when using chequing accounts, ATMs, and

savings accounts.

e. Describe the advantages and disadvantages of online banking, debit card purchases, chequing accounts, and savings accounts.

f. Discuss the use of cheques and determine how to write them.

g. Describe methods taken to ensure the security of personal and financial information (e.g., passwords, encryption, protection of personal identification number (PIN) and other personal identity information) and their effectiveness.

h. Research and discuss various investment options, such as savings accounts, Canada Savings Bonds, Guaranteed Investment Certificates, term investments, RRSPs, and RESPs.

i. Determine simple interest, given three of the four values in the formula $I=Prt$ and explain the reasoning.

j. Determine compound interest using a formula.

k. Compare and contrast simple interest and compound interest.

l. Explain, using examples, the effect of changing different factors on compound interest (e.g., different amortization periods, interest rates, compounding periods, and terms).

m. Estimate, using the Rule of 72, the time required for a given investment to double in value and explain the reasoning.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[FM30.1 and WA30.6]	
M21.10 Demonstrate understanding of financial decision making including analysis of renting, leasing, and buying on credit.	a. Define credit and determine its appropriate use.
	b. Research and discuss various borrowing options, such as credit cards, loans, line of credit, and mortgage.
[C, CN, ME, PS, R, T]	c. Develop an understanding of a credit rating.
	d. Gather and interpret information about credit ratings, and describe the factors used to determine credit ratings and the consequences of a good or bad rating.
	e. Research a variety of credit cards to compare the advantages, disadvantages, and promotions of various credit cards (e.g., financial institutions, store credit cards).
	f. Discuss the effects of carrying an outstanding balance on a credit card at current interest rates.
	g. Identify and compare an installment charge account (e.g., The Brick) or a thirty-day account (e.g., contractor's charge accounts).
	h. Determine, using technology, the total cost of a compound interest loan from various institutions (e.g., banks, payday loans) under a variety of conditions (e.g., different amortization periods, interest rates, compounding periods, and terms).
	i. Compare renting, leasing, and buying of large cost items and generate reasons for considering each choice.

j. Determine the costs related to renting and buying housing.

k. Research and present various options for purchasing or leasing a vehicle (oral, written, multimedia, etc.).

l. Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, and insurance.

m. Solve, with or without technology, questions that involve the purchase, lease, or lease to purchase of a vehicle.

n. Collect and interpret information about the procedures and costs involved in insuring a vehicle (e.g., car, motorcycle, snowmobile) and the factors affecting insurance rates (e.g., gender, age, driving record, model of vehicle, use of vehicle), and compare the insurance costs for different vehicles.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[FM20.1]	
M21.11 Demonstrate understanding of the mathematics involved in an area of interest.	a. Investigate and summarize tourist information around a location of interest (e.g., hours of operation, entry costs, transportation options, travel reviews, safety concerns).
[C, CN, ME, PS, R, T, V]	b. Organize and create a presentation on the chosen location (e.g., pros and cons for visiting, travel brochure, video, guidebook).
	c. Identify and describe situations, experiences, or locations around the area of interest that are relevant to self, family, or community.

d. Compare social justice issues that are present in the location of choice to those present in your community or another community.

e. Identify and explain cultural activities and/or views of mathematics related to the location of interest.

f. Identify and analyze cultural items related to the mathematics at the location of interest.

g. Identify controversial issues or historical events that are or have occurred at the location of interest.

h. Analyze the influences that historically significant events have had on the current field of mathematics.

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes in the provincial curriculum.

Assessment involves the systematic collection of information about student learning with respect to:

- Achievement of provincial curriculum outcomes
- Effectiveness of teaching strategies employed
- Student self-reflection on learning.

Evaluation compares assessment information against criteria based on curriculum outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to curriculum outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Mathematics 21. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the content of Mathematics 21.

Resources

Each theme makes reference to the use of specific websites. Teachers need to consult their board policies regarding use of any copyrighted materials. Before reproducing materials for student use from printed publications, teachers need to ensure that their board has a Can copy licence and that this licence covers the resources they wish to use. Before screening videos/films with their students, teachers need to ensure that their board/school has obtained the appropriate public performance licence. Teachers are reminded that much of the material on the Internet is protected by copyright. The copyright is usually owned by the person or organization that created the work. Reproduction of any work or substantial part of any work on the Internet is not allowed without the permission of the owner.