Math11 (2010)

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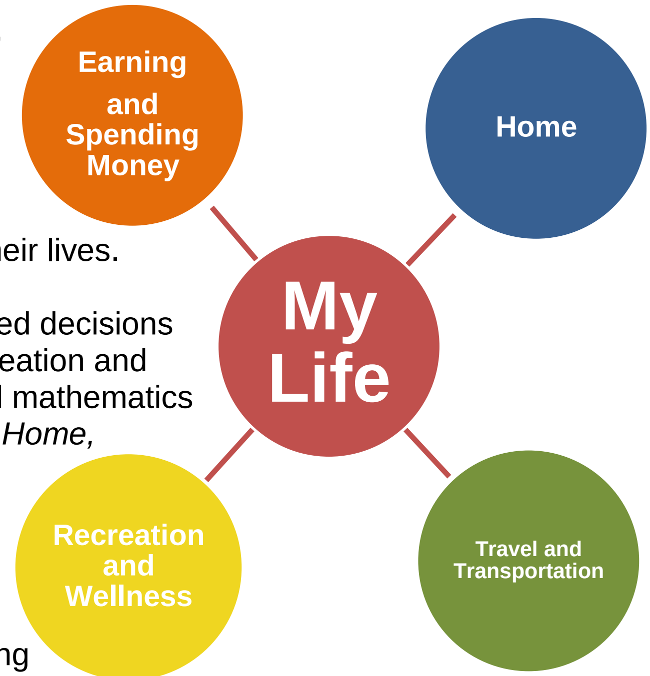
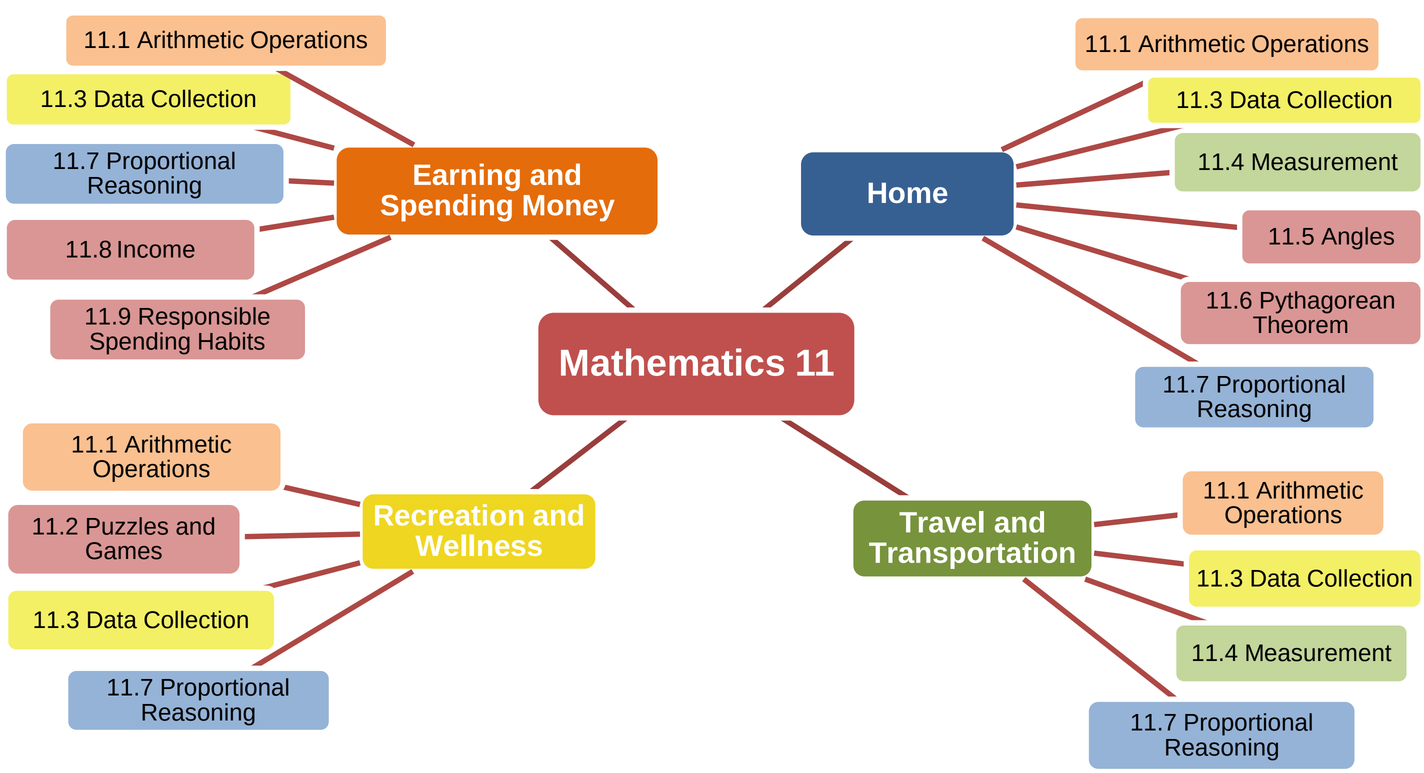
Introduction

Recommended Prerequisite: Mathematics 9



This course is designed for theme-based instruction, which should enable students to broaden their understanding of mathematics as it is applied in important areas of day-to-day living. There is a need for learning to be meaningful in order to be transferable. Learning mathematics should provide students an opportunity to explore mathematics in their lives.

In this course, emphasis is placed on making informed decisions about finances, home design and maintenance, recreation and personal wellness, and travel and transportation. All mathematics relate to the themes: *Earning and Spending Money, Home, Recreation and Wellness,* and *Travel and Transportation.* Students can draw on their own or others experiences in the workforce to develop and extend their knowledge about earning and spending money. They will also apply mathematics for the purpose of designing, building, and maintaining a home and yard. Students will apply reasoning and problem solving skills to make predictions and decisions in recreational and wellness activities. As well, they will investigate and solve problems related to planning a trip.



Adaptive Dimension

In order to meet the variety of students' needs, flexibility is required within the school program to enable schools and teachers to adapt instructional materials, methods, and the environment to provide the most appropriate educational opportunities for students.

The Adaptive Dimension is used to:

- help students achieve curriculum outcomes
- maximize student learning and independence
- lessen discrepancies between achievement and ability
- promote a positive self-image and feeling of belonging
- promote a willingness to become involved in learning
- provide opportunities for all students to be engaged in learning.

These purposes address a primary function of the school, that of helping students to maximize their potentials as independent learners (Ministry of Education, *Core Curriculum Components and Initiatives*, December 17, 2007).

The intent of the Adaptive Dimension applies to all programs and courses of instruction. The key variables of instruction are differentiated – the content (what students will learn), the learning processes (how students will interact with the content), and the learning products (how students will demonstrate learning and mastery of content), and the instructional setting or environment.

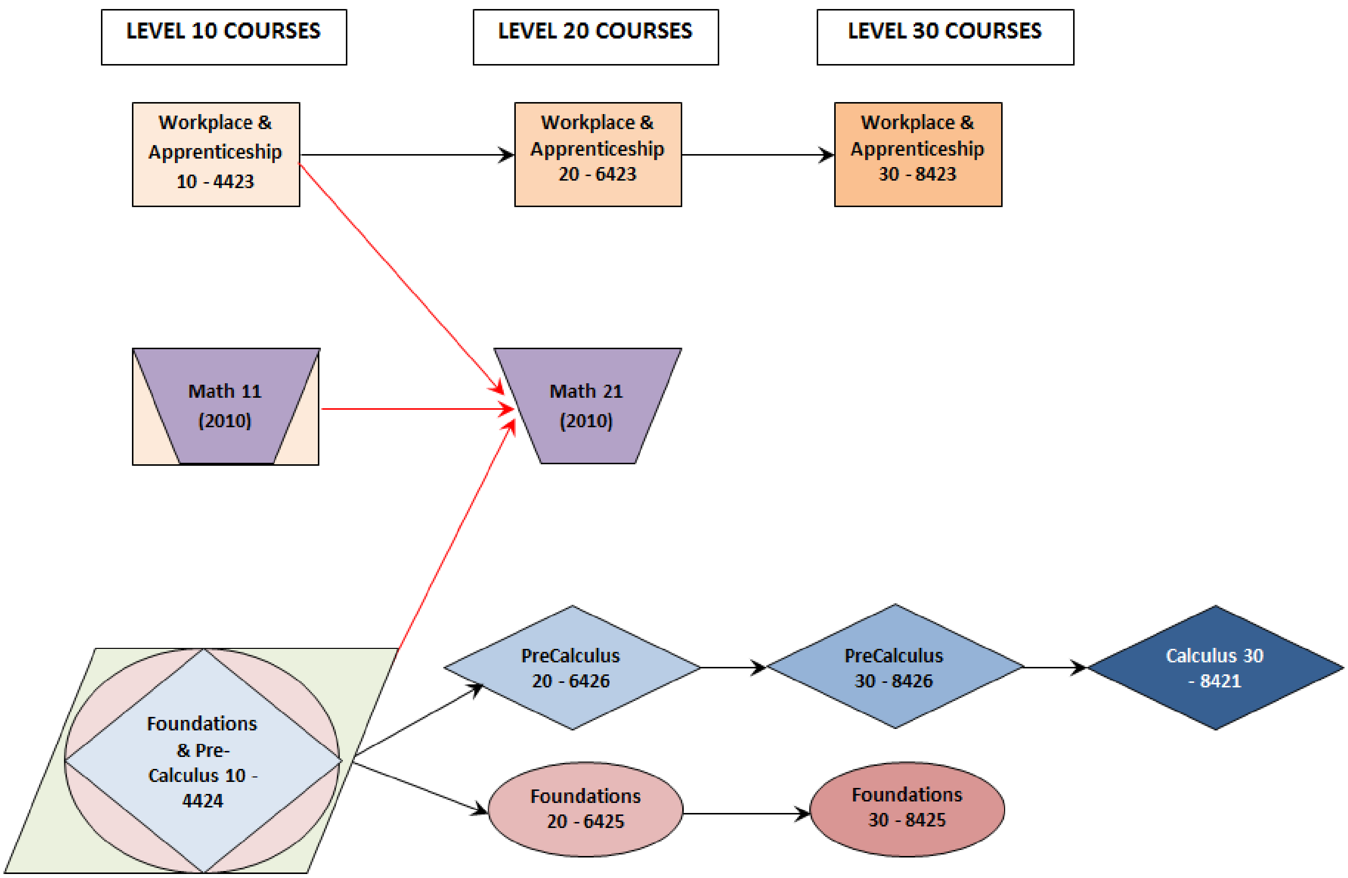
Some students may not be able to complete a Foundations and Pre-calculus 10 and/or Workplace and Apprenticeship 10 course even though adaptations to curriculum materials and topics, instruction, and environment have been made. This may require the development of a modified (Mathematics 11) course to meet student needs to which the Adaptive Dimension may be applied.

Modified Courses

Careful assessment and diagnosis are necessary to understand the language and learning abilities and needs of modified students and to inform instruction. It is important to recognize that the student's academic strengths and weaknesses must be determined through formal and informal means prior to a placement being recommended. If it has been determined through assessment, observation, and collaborative team meetings that a student's needs are best met through placement in a Mathematics 11 course, then all those involved in this decision must carefully consider the implications of such a placement. Everyone including parents/caregivers, students, teachers, and administrators must review the *Policy and Procedures for Locally Modified Courses of Study* with particular attention given to the rationale and implications as outlined. Learning disabilities and/or behaviour disorders should not be the sole criteria for placement in a modified course. Nor, is it appropriate to place students in a modified course because the usual language of instruction is their second language or a non-standard dialect.

Credit Information

Mathematics 11 is a like credit to Workplace & Apprenticeship 10.



→ Pre-requisite

→ Modified pathway

Credit Information:

1. Same colour, vertical difference = like credits
2. Same colour family = general pathway
3. Graduation requirements: 1 credit from the Level 10 column and 1 credit from the level 20 column
   - Level 20 credits are suppressed until an appropriate level 10 credit is granted
   - Level 30 credits are suppressed until appropriate level 10 and 20 credits are granted

Core Curriculum

Core Curriculum is intended to provide all Saskatchewan students with an education that will serve them well regardless of their choices after leaving school. Through its components and initiatives, Core Curriculum supports the achievement of the Goals of Education for Saskatchewan. For current information regarding Core Curriculum, please refer to *Core Curriculum: Principles, Time Allocations, and Credit Policy* (2011) on the Ministry of Education website. For additional information related to the components and initiatives of Core Curriculum, please refer to the Ministry website for various policy and foundation documents.

K-12 Aim and Goals of Mathematics

The K-12 aim of the mathematics program is to have students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities, ongoing learning, and work experiences. The K-12 mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning.

Defined below are four K-12 goals for mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes will therefore, also promote student achievement with respect to the K-12 goals.

Logical Thinking

Through their learning of K-12 mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observing
- inductive and deductive thinking
- proportional reasoning
- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships

- modeling and representing (including concrete, oral, physical, pictorial, and other symbolic representations)
- conjecturing and asking “what if” (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and other symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to be able to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students’ number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation and testing of conjectures based upon patterns that are discovered should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematical and personal)

- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet its particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

All students benefit from mathematics learning which values and respects different ways of knowing mathematics and its relationship to the world. The mathematics content found within this course is often viewed in schools and schooling through a Western or European lens, but there are many different lenses, such as those of many First Nations and Métis peoples, through which mathematics can be viewed and understood. The more exposure that all students have to differing ways of understanding and knowing mathematics, the stronger students will become in their number sense, spatial sense, and logical thinking.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four K-12 goals of mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Critical Characteristics of Mathematics Education

The following sections highlight some of the different facets for teachers to consider in the process of changing from “covering” to supporting students in “discovering” mathematical concepts. These facets include:

- the seven mathematical processes
- the difference between covering and discovering mathematics
- the development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiquing statements
- the concrete to abstract continuum
- modelling and making connections
- the role of homework
- the importance of ongoing feedback and reflection.

Mathematical Processes

This Mathematics 11 course recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing, along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding.

The outcomes in mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. During planning, teachers should carefully consider those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, prior knowledge, the formal language and symbols of mathematics, and new learning.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding of that terminology.

Concrete, pictorial, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase students' willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. When estimating, students need to know what strategy to use, when to use it, and how to use it.

Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you ...?”, “Can you ...?”, or “What if ...?”, the problem solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.
In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students are given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. Meaningful inquiry challenges students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Technology [T]

Technology tools contribute to student achievement of a wider range of mathematics outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what must be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols and quantities.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies, processes, and rules, as well as the students' current and intuitive understandings of quantity, patterns, and shapes. Any learning in mathematics that is a consequence of the logical structure of mathematics can and should be constructed by students.

Development of Mathematical Terminology

Part of learning mathematics is learning how to communicate mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that are already known or that make sense to students.

First Nations and Métis Learners and Mathematics

Teachers must recognize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understanding. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it is not taught in relation to their schema, cultural and environmental context, or real life experiences.

A first step in the actualization of mathematics from First Nations and Métis perspectives is empowering teachers to understand that mathematics is not acultural. As a result, teachers realize that the traditional Western European ways of teaching mathematics also are culturally biased. These understandings will support the teacher in developing First Nations and Métis students' personal mathematical understanding and mathematical self-confidence and ability through a more holistic and constructivist approach to teaching. Teachers need to pay close attention to those factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

Teachers must recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding, and as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas among cultural groups cannot be assumed to be a direct link. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematics classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. In addition, they also allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. As well, ethnomathematics demonstrates the relationship between mathematics and cultural anthropology.

Individually and as a class, teachers and students need to explore the big ideas that are foundational to this course and investigate how those ideas relate to themselves personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of Elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthens the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in students' understanding. Some indicators in this course are examples of statements that students can analyze for accuracy.

Critiquing statements is an effective way to assess students individually, as a small group, or as a large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important, in learning mathematics, that students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a more concrete starting point. Therefore, teachers must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness.

Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and other symbolic models). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you represent what you've done here using mathematical symbols?) or by having students compare their representations with others in the class. In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help to consolidate new learnings with prior knowledge, explore possible solutions, and apply learning to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of drill will vary among different learners. In addition, when used as homework, drill and practice frequently causes frustration, misconceptions, and boredom to arise in students.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information to teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflecting
- exploring of patterns and relationships
- sharing ideas and problems
- considering different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to course content and outcomes.

Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are involved and engaged directly in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning of course content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with parts of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge.

Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, develop conclusions, document and reflect on learning, and generate new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students must understand this difference too.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more entry points; however, the process may evolve into transdisciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry, and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or transdisciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "How do you know when you have an answer?"
- "Will this strategy work for all situations?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning. Questioning should also be used to encourage students to reflect on the inquiry process and on the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- help students make sense of the mathematics.
- are open-ended, whether in answer or approach, as there may be multiple answers or multiple approaches.
- empower students to unravel their misconceptions.
- not only require the application of facts and procedures but encourage students to make connections and generalizations.
- are accessible to all students and offer an entry point for all students.
- lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.

(Schuster & Canavan Anderson, 2005, p. 3)

Organization of Outcomes and Themes

For ease of reference, the outcomes in this document are numbered using the following system M11.#, where # is the number of the outcome in the list of outcomes. It should be noted, for example, that M11.1 need not be taught before M11.2.

There are four themes in this course, *Earning and Saving Money, Home, Recreation and Wellness,* and *Travel and Transportation*. The ordering and grouping of the themes in Mathematics 11 is at the discretion of the teacher. Teachers are encouraged to design learning activities that integrate outcomes from throughout the course so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate topics.

Introduction of Outcomes and Indicators

The outcomes in the Mathematics 11 course are based upon the students' prior learning and continue to develop their number sense, spatial sense, logical thinking, and understanding of mathematics as a human endeavour. These learning experiences prepare students to be confident, flexible, and capable with their mathematical knowledge in new contexts. The outcomes in this course are the prerequisite outcomes for the Mathematics 21 course.

Outcomes describe the knowledge, skills, and understandings that students are expected to attain by the end of a particular grade. The mathematical knowledge and skills acquired through this course will be useful to students in many applications throughout their lives in both work and non-work settings.

Indicators are included for each of the outcomes in order to clarify the breadth and depth of learning intended by the outcome. New and combined indicators, which remain within the breadth and depth of the outcome, can and should be created by teachers to meet the needs and circumstances of their students and communities.

The Mathematics 11 outcomes have included some outcomes and indicators from the Foundations and Pre-calculus 10 and Workplace and Apprenticeship Mathematics 10 outcomes. The outcomes and indicators written in **green font** are from the current Foundations and Pre-calculus 10 or Workplace and Apprenticeship Mathematics 10 outcomes; the additional outcomes and indicators written in **black font** are unique to the Mathematics 11 course. The intent of the different colours of font is to assist teachers who teach combined classes.

Outcomes and Indicators

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
M11.1 Extend understanding of arithmetic operations to rational numbers to solve problems within the home, money, recreation, and travel themes. **[C, CN, ME, R, V]**	a. Compare and order positive and negative numbers, using appropriate tools (e.g., change in temperature using a thermometer). b. Apply arithmetic operations to whole numbers, integers, fractions, decimals, and percents. c. Compare and convert among fractions, decimals and percents concretely, pictorially, and symbolically. d. Determine rounding of decimals to the nearest unit, tenth and hundredth (e.g., calculations with money rounding to 2 decimal places). e. Apply understanding of combined percents or percents of percents. f. Justify the reasonableness of calculations and problem-solving strategies, using a variety of tools and/or strategies (e.g., estimation, mental mathematics, tables, graphs, calculators and/or computers).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA 10.2]	
M11.2 Demonstrate understanding of reasoning by analyzing puzzles and games. [C,CN,PS,R,V]	a. Determine, explain and verify strategies to solve a puzzle or win a game such as: • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • solve a simpler problem • work backwards • develop alternative approaches. b. Analyze puzzles or games for patterns, describe the properties of a given pattern, and identify if a set of objects fits the pattern or not and explain why. c. Observe and analyze errors in a solution to a puzzle or in a strategy for winning a game and explain the reasoning. d. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game. ***Sample games:*** *Tetris, Rubik's cube, Blokus, chess, checkers, Backgammon, Mastermind, Tic-Tac-Toe, Connect Four or Five, Battleship, Cathedral World, and Mancala.*

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
M11.3 Demonstrate understanding of data collection and analysis within the home, recreation, and travel themes. **[C, CN, PS, R, V, T]**	a. Read and interpret graphs (e.g., line graph, broken-line graph, bar graph, histogram, circle graph) obtained from various sources (e.g., newspapers, magazines, Statistics Canada website) and communicate information represented in relation to real life situations (e.g., weight, weight training programs, box scores for sports, nutrition, sleep, physical activity, sporting events, and travel). b. Design questionnaires (e.g., for a cafeteria to determine which juices to stock) or experiments (e.g., observing, counting, taking measurements) for gathering data. c. Explain the difference between population and sample, describe the characteristics of a good sample, and explain why sampling is necessary (e.g., time, cost, or physical constraints). d. Collect data from primary sources (e.g., surveys, questionnaires, experiments, interviews) or from secondary sources (e.g., Internet databases, newspapers, magazines). e. Organize data from primary or secondary sources in a variety of ways (e.g. table, frequency table, stem and leaf plot). f. Represent data using graphs (e.g., line graph, broken-line graph, bar graph, histogram, circle graph) using a variety of tools (e.g., dynamic statistical software, graphing calculator, spreadsheet) and justify the type of graph chosen. g. Evaluate graphical representations of data through inferences, comparisons, and predictions and justify conclusions using convincing arguments.

h. Describe and analyze situations in which data has been collected (e.g., target heart rates, weather patterns and predictions, sports scores).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA 10.3, WA 10.5, FP 10.3] **M11.4 Demonstrate understanding of measurement in the Système International (metric) and Imperial System within the home and travel themes.** **[C, CN, ME, T, V]**	*It is intended that students explore, analyze for patterns, and develop understanding of many units in the systems of measurements. The units used should be those that are appropriate to the context being considered. These units include:* • *metres, grams, litres, and seconds along with appropriate prefixes such as kilo, centi, and milli, and degrees Celsius (SI system).* • *inch, foot, mile, teaspoon, tablespoon, cup, pint, quart, gallon, and degrees Fahrenheit (Imperial system).* a. Determine and explain the lengths of common objects in the metric and imperial systems, using a variety of tools (e.g., measuring tape, metre or yard stick, measuring cups, graduated cylinders, trundle wheel). b. Estimate lengths and distances in metric units and in imperial units by applying personal referents (e.g., the width of a finger is approximately 1 cm; the length of a piece of standard loose-leaf paper is about 1 ft; the capacity of a pop bottle is 2 L). c. Develop, explain, and apply strategies to estimate quantities (e.g., books in a shelving unit, time to complete a job, people in a crowd). d. Determine and explain the mathematics related to time including: • converting units of time measure • 24-hour clock

- time zones
- flight arrival and departure times
- elapsed time.

e. Convert measures within and between systems (e.g., centimeters and metres, feet and inches, pounds and kilograms, degrees Celsius and degrees Fahrenheit) using a variety of tools (e.g., tables, calculators, online conversion tools).

f. Discuss and approximate measures between systems (e.g., 1 inch is approximately 2.5 cm, 1 kg is a little more than 2 lbs, 1 litre is approximately $\frac{1}{4}$ US gallon).

g. Describe the situations in which SI and/or Imperial units of measurement are used.

h. Analyze the relationships between the related units for length, area, temperature, and currency measures within and between systems.

i. Estimate, measure, and calculate perimeters (e.g., wall paper borders, fencing, baseboards).

j. Estimate, measure, and calculate areas of rectangles, triangles, circles, and of related composite shapes (e.g., wall space to be painted, floors to be covered, square footage of a living space, laying sod, patio slabs, floor tiles, wall paper).

k. Critique the statement “the distance between Regina and Saskatoon is 2 hours”. (WA10.4)

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes

[WA 10.9]

M11.5 Demonstrate understanding of angles to solve problems within the home theme.

[C, ME, PS, T, V]

Indicators

a. Justify the choice of personal referents for angles measuring 22.5°, 30°, 45°, 60°, 90°, and 180° and use them to estimate angle measurements (e.g., a corner of a sheet of paper is 90° so ½ of a corner is 45°).

b. Explain, using home construction examples (e.g., mitre cuts, framing, window and door casings, trusses, tile installations, crown moulding), how to measure angles in different orientations using a variety of instruments (e.g., protractor, carpenter's square, and dynamic software).

c. Explain and illustrate how angles can be replicated and drawn (e.g., Mira, protractor, compass and straightedge, carpenters square and dynamic software).

d. Identify, classify, and sketch angles of various measures, including acute, right, straight, obtuse, and reflex angles.

e. Explain, using examples, the relationship between the bisecting of angles and axial symmetry.

f. Bisect angles in various orientations and explain the strategy used.

g. Identify adjacent angles that are complementary, supplementary, or neither, and explain the reasoning.

h. Solve situational problems involving complementary and supplementary angles.

i. Identify vertically opposite angles and solve situational problems.

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	**Indicators**
[WA 10.6]	
M11.6 Demonstrate understanding of the Pythagorean Theorem to solve problems within the home theme. **[C, CN, PS, V, T]**	a. Model, including the use of drawings, concrete materials, and technology, the meaning, role, and use of the Pythagorean Theorem, using examples and non-examples. b. Apply the Pythagorean 3:4:5 ratios to determine if angles are square (right angles) in home construction contexts. c. Apply the Pythagorean Theorem to solve for a missing side that has an irrational solution. d. Estimate the values of irrational numbers using a table of perfect squares, multiplication chart, or a number line and show appropriate rounding of irrational numbers. e. Observe and analyze the use of Pythagorean lengths of diagonals of various building structures (e.g., trusses, frames, door jambs, window casings).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA 10.10]	
M11.7 Demonstrate understanding of proportional reasoning within the home, money, recreation, and travel themes. **[CN, ME, PS, R, T]**	a. Explain and apply strategies to solve ratio and rate problems. b. Recognize and represent equivalent rates and ratios. c. Calculate and compare the unit rate of items and the unit cost of items (e.g., heart rates in various situations, walking speed, rate of pay, cost per linear foot). d. Calculate and compare costs of items (e.g., lodgings, transportation, recreation fees, cellular mobile phone plans). e. Estimate and calculate conversions between Canadian and American currency using proportional reasoning. f. Identify and describe applications of proportional reasoning (e.g., applying fertilizers, mixing gasoline and oil for use in small engines, estimating cooking time needed per pound, determining the fiber content of different sizes of food servings, calculating overtime pay).

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
[WA 10.11]	
M11.8 Demonstrate understanding of income. **[C, CN, R, T]**	a. Gather, interpret, and describe various remuneration methods of earning income (e.g., hourly rate, overtime rate, job or project rate, commission, salary, gratuities) and remuneration schedules (e.g., weekly, biweekly, semimonthly, monthly). b. Research and record jobs that commonly use different remuneration methods of earning income (e.g., hourly wage, wage and tips, salary, commission, contract, piecework, bonus, and shift premiums). c. Describe the advantages and disadvantages for various remuneration methods of earning income (e.g., hourly wage, tips, piecework, salary, commission, contract work, and self-employment). d. Solve problems and make decisions involving different remuneration methods and schedules. e. Analyze and complete timesheets. f. Explain and assess the information provided on pay stubs. g. Determine gross pay for different situations, including base hourly wage, with and without tips, from given or calculated hours worked base hourly wage, plus overtime (time and a half, double time) from given or calculated hours worked base wage, plus commission, single commission rate and graduated commission. h. Gather, interpret, and describe information about government payroll deductions (e.g., CPP, EI, income tax) and other payroll deductions (e.g., contributions to pension plans other than CPP; union dues; charitable donations; benefit-plan

contributions).

i. Estimate and compare, using current data (e.g., federal tax tables), the percent of total earnings deducted through government payroll deductions for various benchmarks (e.g., $15 000, $20 000, $25 000).

j. Describe the relationship between gross pay, net pay, and payroll deductions (e.g., net pay is gross pay less government payroll deductions and any other payroll deductions), and estimate net pay in various situations.

k. Investigate, with or without technology, "what if …" questions related to changes in income (e.g., "What if there is a change in the rate of pay?", "What if there is a change in the method of earning income?", "What if I can qualify for deductions?", "What if I work 80% time instead of full time?", "What if I am sick for a long period of time?", "What if an athlete earned one million dollars last year, then how many hours would I have to work to earn that much money?").

Goals: Number Sense, Logical Thinking, Spatial Sense, Mathematics as a Human Endeavour

Outcomes	Indicators
M11.9 Demonstrate understanding of responsible spending habits. **[C, CN, ME, PS, R, T]**	a. Identify and justify personal expenses (e.g., mobile phone, vehicle, electronics, recreation, travel, home renovations, and aesthetics). b. Explain considerations made when prioritizing spending money (e.g., recurring expenses and unexpected opportunities). c. Estimate the cost and justify affordability of a desired purchase. d. Determine PST and GST on purchases and discuss exemptions. e. Create a personal spending log over a set period

of time and explain the advantages.

f. Compare and contrast the cost of purchasing items or services at various vendors.

g. Compare various sales incentives (e.g., “Group on”, percent discounts, pre-sale gift with purchase, reward zone points, buy 1, and get 1 . . . (BOGO)) and discuss the value of estimating.

h. Investigate and analyze “what if …” questions related to personal spending (e.g., cash versus debit purchases involving the penny or rounding).

i. Research and report on the estimated costs involved in a large expense (e.g., a trip, home renovation, or an activity or sport).

Assessment and Evaluation of Student Learning

Assessment and evaluation require thoughtful planning and implementation to support the learning process and to inform teaching. All assessment and evaluation of student achievement must be based on the outcomes.

Assessment involves the systematic collection of information about student learning with respect to:

- Achievement of outcomes
- Effectiveness of teaching strategies employed
- Student self-reflection on learning.

Evaluation compares assessment information against criteria based on outcomes for the purpose of communicating to students, teachers, parents/caregivers, and others about student progress and to make informed decisions about the teaching and learning process.

Reporting of student achievement must be in relation to outcomes. Assessment information which is not related to outcomes can be gathered and reported (e.g., attendance, behaviour, general attitude, completion of homework, effort) to complement the reported achievement related to the outcomes of Mathematics 11. There are three interrelated purposes of assessment. Each type of assessment, systematically implemented, contributes to an overall picture of an individual student's achievement.

Assessment for learning involves the use of information about student progress to support and improve student learning and inform instructional practices, and:

- is teacher-driven for student, teacher, and parent use
- occurs throughout the teaching and learning process, using a variety of tools
- engages teachers in providing differentiated instruction, feedback to students to enhance their learning, and information to parents in support of learning.

Assessment as learning involves student reflection on and monitoring of her/his own progress related to curricular outcomes and:

- is student-driven with teacher guidance for personal use
- occurs throughout the learning process
- engages students in reflecting on learning, future learning, and thought processes (metacognition).

Assessment of learning involves teachers' use of evidence of student learning to make judgements about student achievement and:

- provides opportunity to report evidence of achievement related to curricular outcomes
- occurs at the end of a learning cycle, using a variety of tools
- provides the foundation for discussion on placement or promotion.

In mathematics, students need to be regularly engaged in assessment as learning. The various types of assessments should flow from the learning tasks and provide direct feedback to the students regarding their progress in attaining the desired learnings as well as opportunities for the students to set and assess personal learning goals related to the content of Mathematics 11.

Resources

Each theme makes reference to the use of specific websites. Teachers need to consult their board policies regarding use of any copyrighted materials. Before reproducing materials for student use from printed publications, teachers need to ensure that their board has a Can copy licence and that this licence covers the resources they wish to use. Before screening videos/films with their students, teachers need to ensure that their board/school has obtained the appropriate public performance licence. Teachers are reminded that much of the material on the Internet is protected by copyright. The copyright is usually owned by the person or organization that created the work. Reproduction of any work or substantial part of any work on the Internet is not allowed without the permission of the owner.