Good Spirit School Division

Mathematics 18

June 2015

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Math 18 Introduction

(taken from the Grade 9 Math Saskatchewan Online Curriculum)

Broad Areas of Learning

There are three Broad Areas of Learning that reflect Saskatchewan's Goals of Education. K-12 mathematics contributes to the Goals of Education through helping students achieve knowledge, skills, and attitudes related to these Broad Areas of Learning.

Developing Lifelong Learners

Students who are engaged in constructing and applying mathematical knowledge naturally build a positive disposition towards learning. Throughout their study of mathematics, students should be learning the skills (including reasoning strategies) and developing the attitudes that will enable the successful use of mathematics in daily life. Moreover, students should be developing understandings of mathematics that will support their learning of new mathematical concepts and applications that may be encountered within both career and personal interest choices. Students who successfully complete their study of K-12 mathematics should feel confident about their mathematical abilities and have developed the knowledge, understandings, and abilities necessary to make future use and/ or studies of mathematics meaningful and attainable.

In order for mathematics to contribute to this Broad Area of Learning, students must actively learn the mathematical content in the outcomes through using and developing logical thinking, number sense, spatial sense, and understanding of mathematics as a human endeavour (the four goals of K-12 Mathematics). It is crucial that the students discover the mathematics outlined in the curriculum rather than the teacher covering it.

Developing a Sense of Self and Community

To learn mathematics with deep understanding, students not only need to interact with the mathematical content, but with each other as well. Mathematics needs to be taught in a dynamic environment where students work together to share and evaluate strategies and understandings. Students who are involved in a supportive mathematics learning environment that is rich in dialogue are exposed to a wide variety of perspectives and strategies from which to construct a sense of the mathematical content. In such an environment, students also learn and come to value how they, as individuals and as members of a group or community, can contribute to understanding and social well-being through a sense of accomplishment, confidence, and relevance. When encouraged to present ideas representing different perspectives and ways of knowing, students in mathematics classrooms develop a deeper understanding of the mathematics. At the same time, students also learn to respect and value the contributions of others.

Mathematics provides many opportunities for students to enter into communities beyond the classroom by engaging with people in the neighbourhood or around the world. By working towards developing a deeper understanding of mathematics and its role in the world, students develop their personal and social identity, and learn healthy and positive ways of interacting and working together with others.

Developing Engaged Citizens

Mathematics brings a unique perspective and way of knowing to the analysis of social impact and interdependence. Doing mathematics requires students to "leave their emotions at the door" and to engage in different situations for the purpose of understanding what is really happening and what can be done. Mathematical analysis of topics that interest students such as trends in climate change, homelessness, health issues (hearing loss, carpal tunnel syndrome, diabetes), and discrimination can be used to engage the students in interacting and contributing positively to their classroom, school, community, and world. With the understandings that students derive through mathematical analysis, they become better informed and have a greater respect for and understanding of differing opinions and possible options. With these understandings, students can make better informed and more personalized decisions regarding roles within, and contributions to, the various communities in which students are members.

Cross-curricular Competencies

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes which are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking

It is important that, within their study of mathematics, students are engaged in personal construction and understanding of mathematical knowledge. This most effectively occurs through student engagement in inquiry and problem solving when students are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which students are asked to consider questions such as "What would happen if …", "Could we find …", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool which can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence

Given an appropriate learning environment in mathematics, students can develop both their self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It can also help students take an active role in defining and maintaining the classroom environment and accept

responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning serves to support students in behaving respectfully towards themselves and others.

Developing Literacies

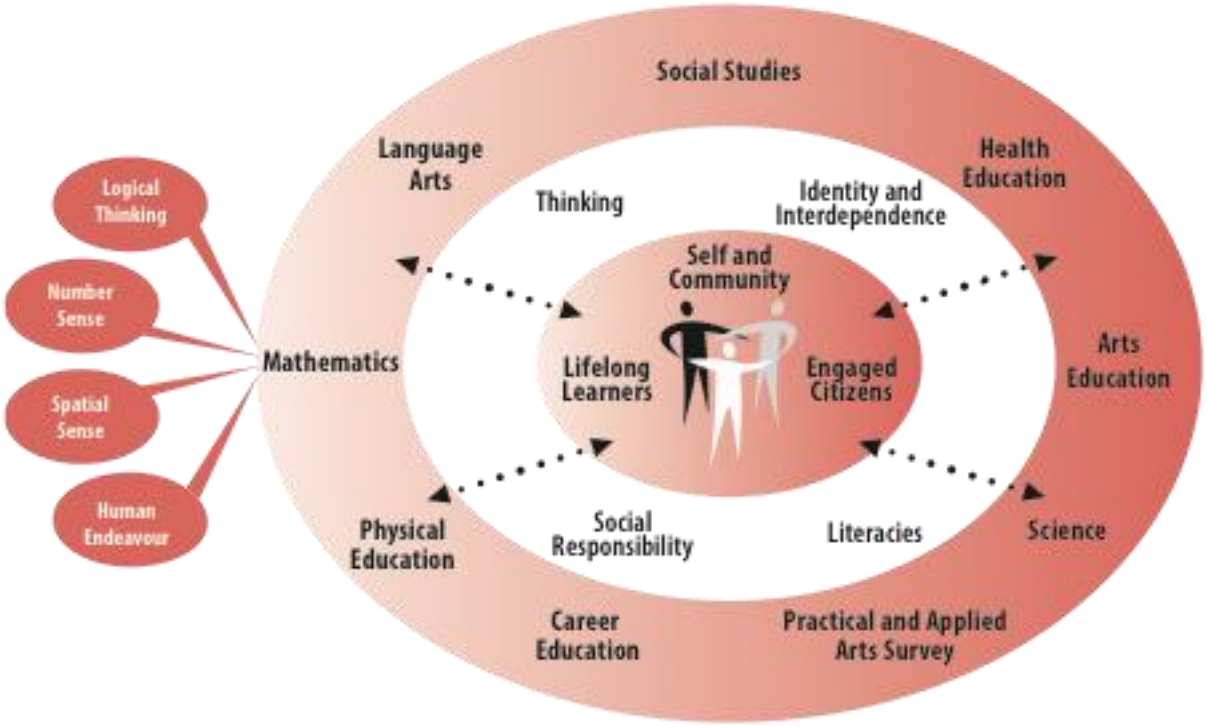
Through their mathematics learning experiences, students should be engaged in developing their understandings of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be regularly engaged in exploring a variety of representations for mathematical concepts and should be expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language is to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. The study of mathematics should encourage the appropriate use of technology. Moreover, students should be aware of and able to make the appropriate use of technology in mathematics and mathematics learning. It is important to encourage students to use a variety of forms of representation (concrete manipulatives, physical movement, oral, written, visual, and symbolic) when exploring mathematical ideas, solving problems, and communicating understandings. All too often, it is assumed that symbolic representation is the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper students' understanding becomes.

Developing Social Responsibility

As student's progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires that the learning environment be constructed by the teacher and students to support respectful, independent, and interdependent behaviours. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, students can be engaged in understanding the situation, concern, or issue and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for situational validity, and strengthened.

Aims and Goals of K-12 Mathematics

The aim of Saskatchewan's K-12 mathematics program is to help students develop the understandings and abilities necessary to be confident and competent in thinking and working mathematically in their daily activities and ongoing learnings and work experiences. The mathematics program is intended to stimulate the spirit of inquiry within the context of mathematical thinking and reasoning. Defined below are four goals for K-12 mathematics in Saskatchewan. The goals are broad statements that identify the characteristics of thinking and working mathematically. At every grade level, students' learning should be building towards their attainment of these goals. Within each grade level, outcomes are directly related to the development of one or more of these goals. The instructional approaches used to promote student achievement of the grade level outcomes must, therefore, also promote student achievement with respect to the goals.



Logical Thinking

Through their learning of K-12 Mathematics, students will **develop and be able to apply mathematical reasoning processes, skills, and strategies to new situations and problems.**

This goal encompasses processes and strategies that are foundational to understanding mathematics as a discipline. These processes and strategies include:

- observation
- inductive and deductive thinking
- proportional reasoning

- abstracting and generalizing
- exploring, identifying, and describing patterns
- verifying and proving
- exploring, identifying, and describing relationships
- modeling and representing (including concrete, oral, physical, pictorial, and symbolic representations)
- conjecturing and asking "what if" (mathematical play).

In order to develop logical thinking, students need to be actively involved in constructing their mathematical knowledge using the above strategies and processes. Inherent in each of these strategies and processes is student communication and the use of, and connections between, multiple representations.

Number Sense

Through their learning of K-12 mathematics, students will **develop an understanding of the meaning of, relationships between, properties of, roles of, and representations (including symbolic) of numbers and apply this understanding to new situations and problems.**

Foundational to students developing number sense is having ongoing experiences with:

- decomposing and composing of numbers
- relating different operations to each other
- modeling and representing numbers and operations (including concrete, oral, physical, pictorial, and symbolic representations)
- understanding the origins and need for different types of numbers
- recognizing operations on different number types as being the same operations
- understanding equality and inequality
- recognizing the variety of roles for numbers
- developing and understanding algebraic representations and manipulations as an extension of numbers
- looking for patterns and ways to describe those patterns numerically and algebraically.

Number sense goes well beyond being able to carry out calculations. In fact, in order for students to become flexible and confident in their calculation abilities, and to transfer those abilities to more abstract contexts, students must first develop a strong understanding of numbers in general. A deep understanding of the meaning, roles, comparison, and relationship between numbers is critical to the development of students' number sense and their computational fluency.

Spatial Sense

Through their learning of K-12 mathematics, students will **develop an understanding of 2-D shapes and 3-D objects, and the relationships between geometrical shapes and objects and numbers, and apply this understanding to new situations and problems.**

Development of a strong spatial sense requires students to have ongoing experiences with:

- construction and deconstruction of 2-D shapes and 3-D objects
- investigations and generalizations about relationships between 2-D shapes and 3-D objects
- explorations and abstractions related to how numbers (and algebra) can be used to describe 2-D shapes and 3-D objects
- explorations and generalizations about the movement of 2-D shapes and 3-D objects
- explorations and generalizations regarding the dimensions of 2-D shapes and 3-D objects
- explorations, generalizations, and abstractions about different forms of measurement and their meaning.

Being able to communicate about 2-D shapes and 3-D objects is foundational to students' geometrical and measurement understandings and abilities. Hands-on exploration of 3-D objects and the creation of conjectures based upon patterns that are discovered and tested should drive the students' development of spatial sense, with formulas and definitions resulting from the students' mathematical learnings.

Mathematics as a Human Endeavour

Through their learning of K-12 mathematics, students will **develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs.**

Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:

- value place-based knowledge and learning
- value learning from and with community
- encourage and value varying perspectives and approaches to mathematics
- recognize and value one's evolving strengths and knowledge in learning and doing mathematics
- recognize and value the strengths and knowledge of others in doing mathematics
- value and honour reflection and sharing in the construction of mathematical understanding
- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics.

Students should be encouraged to challenge the boundaries of their experiences, and to view mathematics as a set of tools and ways of thinking that every society develops to meet their particular needs. This means that mathematics is a dynamic discipline in which logical thinking, number sense, and spatial sense form the backbone of all developments and those developments are determined by the contexts and needs of the time, place, and people.

The content found within the grade level outcomes for the K-12 mathematics program, and its applications, is first and foremost the vehicle through which students can achieve the four goals of K-12 mathematics. Attainment of these four goals will result in students with the mathematical confidence and tools necessary to succeed in future mathematical endeavours.

Teaching Mathematics

At the National Council of Teachers of Mathematics (NCTM) Canadian Regional Conference in Halifax (2000), Marilyn Burns said in her keynote address, "When it comes to mathematics curricula there is very little to cover, but an awful lot to uncover [discover]." This statement captures the essence of the ongoing call for change in the teaching of mathematics. Mathematics is a dynamic and logic-based language that students need to explore and make sense of for themselves. For many teachers, parents, and former students, this is a marked change from the way mathematics was taught to them. Research and experience indicate there is a complex, interrelated set of characteristics that teachers need to be aware of in order to provide an effective mathematics program.

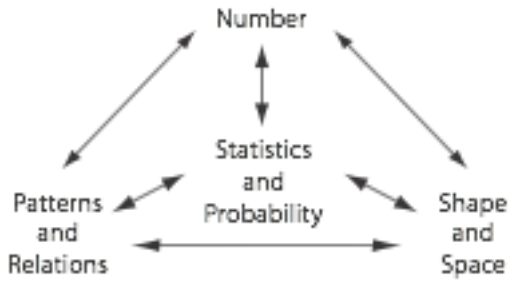
Critical Characteristics of Mathematics Education

The following sections in this curriculum highlight some of the different facets for teachers to consider in the process of changing from covering to supporting students in discovering mathematical concepts. These facets include:

- organization of the outcomes into strands
- seven mathematical processes
- the difference between covering and discovering mathematics
- development of mathematical terminology
- First Nations and Métis learners and mathematics
- critiqueing statements
- continuum of understanding from concrete to abstract
- modelling and making connections
- role of homework
- importance of ongoing feedback and reflection.

Strands

The content of K-12 mathematics can be organized in a variety of ways. In this curriculum, the outcomes and indicators are grouped according to four strands: **Number, Patterns and Relations, Shape and Space, and Statistics and Probability.** Although this organization implies a relatedness among the outcomes identified in each of the strands, it should be noted the mathematical concepts are interrelated across the strands as well as within strands. Teachers are encouraged to design learning activities that integrate outcomes both within a strand and across the strands so that students develop a comprehensive and connected view of mathematics rather than viewing mathematics as a set of compartmentalized ideas and separate strands.



Mathematical Processes

This Grade 9 Mathematics curriculum recognizes seven processes inherent in the teaching, learning, and doing of mathematics. These processes focus on: communicating, making connections, mental mathematics and estimating, problem solving, reasoning, and visualizing along with using technology to integrate these processes into the mathematics classroom to help students learn mathematics with deeper understanding. The outcomes in K-12 mathematics should be addressed through the appropriate mathematical processes as indicated by the bracketed letters following each outcome. Teachers should consider carefully in their planning those processes indicated as being important to supporting student achievement of the respective outcomes.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas using both personal and mathematical language and symbols. These opportunities allow students to create links among their own language, ideas, and prior knowledge, the formal language and symbols of mathematics, and new learnings.

Communication is important in clarifying, reinforcing, and adjusting ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology, but only when they have had sufficient experience to develop an understanding for that terminology.

Concrete, pictorial, symbolic, physical, verbal, written, and mental representations of mathematical ideas should be encouraged and used to help students make connections and strengthen their understandings.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to other real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

The brain is constantly looking for and making connections. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and prior knowledge, and increase student willingness to participate and be actively engaged.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally and reasoning about the relative size of quantities without the use of external memory aids. Mental mathematics enables students to determine answers and propose strategies without paper and pencil. It improves computational fluency and problem solving by developing efficiency, accuracy, and flexibility.

Estimation is a strategy for determining approximate values of quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgements and develop useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you …?”, “Can you …?”, or “What if …?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not problem solving but practice. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple and creative solutions. Creating an environment where students actively look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confidence, reasoning, and mathematical creativity.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and explain their mathematical thinking. High-order inquiry challenges students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom should provide opportunities for students to engage in inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Visualization [V]

The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number sense, spatial sense, and logical thinking. Number visualization occurs when students create mental representations of numbers and visual ways to compare those numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes including aspects such as dimensions and measurements.

Visualization is also important in the students' development of abstraction and abstract thinking and reasoning. Visualization provides a connection between the concrete, physical, and pictorial to the abstract symbolic. Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations as well as the use of communication to develop connections among different contexts, content, and representations.

Technology [T]

Technology tools contribute to student achievement of a wide range of mathematical outcomes, and enable students to explore and create patterns, examine relationships, test conjectures, and solve problems. Calculators, computers, and other forms of technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense
- develop spatial sense
- develop and test conjectures.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. It is important for students to understand and appreciate the appropriate use of technology in a mathematics classroom. It is also important that students learn to distinguish between when technology is being used appropriately and when it is being used inappropriately. Technology should never replace understanding, but should be used to enhance it.

Discovering versus Covering

Teaching mathematics for deep understanding involves two processes: teachers covering content and students discovering content. Knowing what needs to be covered and what can be discovered is crucial in planning for mathematical instruction and learning. The content that needs to be covered (what the teacher needs to explicitly tell the students) is the social conventions or customs of mathematics. This content includes things such as what the symbol for an operation looks like, mathematical terminology, and conventions regarding recording of symbols.

The content that can and should be discovered by students is the content that can be constructed by students based on their prior mathematical knowledge. This content includes things such as strategies and procedures, rules, and problem solving. Any learning in mathematics that is a result of the logical structure of mathematics can and should be constructed by students.

For example, in Grade 9, the students encounter similarity of 2-D shapes for the first time in outcome SS9.3 :

> Demonstrate understanding of similarity of 2-D shapes.
>
> [C, CN, PS, R, V]

In this outcome, the term “similarity” and the symbol “~” are both social conventions of the mathematics the students are learning and, as such, both are something that the teacher must tell the student. Identifying and describing patterns of proportionality between the side lengths of 2-D shapes as well as the relationship between interior angles is the foundation of the students’ construction of understanding. This type of learning requires students to work concretely, pictorially, orally, in writing, and symbolically. It also requires that students share their ideas with classmates and reflect upon how the ideas and understandings of others relate to, inform, and clarify what students individually understand. In this type of learning, the teacher does not tell the students how to do the mathematics but, rather, invites the students to explore and develop an understanding of the logical structures inherent in the mathematics in increasing patterns. Thus, the teacher’s role is to create inviting and rich inquiring tasks and to use questioning to effectively probe and further students’ learning.

Development of Mathematical Terminology

Part of learning mathematics is learning how to speak mathematically. Teaching students mathematical terminology when they are learning for deep understanding requires that the students connect the new terminology with their developing mathematical understanding. As a result, it is important that students first linguistically engage with new mathematical concepts using words that students already know or that make sense to them.

For example, in outcome SS9.1:

Demonstrate understanding of circle properties including:

- perpendicular line segments from the centre of a circle to a chord bisect the chord
- inscribed angles subtended by the same arc have the same measure
- the measure of a central angle is twice the measure of an inscribed angle subtending the same arc
- tangents to a circle are perpendicular to the radius ending at the point of tangency.

[C, CN, PS, R, T, V]

the terminology of “chord”, “subtended”, “inscribed angles”, “arc”, “central angle”, and “point of tangency” will likely be new. Before being formally introduced to this terminology, it is important that students be concretely and pictorially, including through the use of technology, representing and analyzing different relationships involving lines, angles, and circles. As the students discover different relationships, their need for specific terminology emerges and it is at that time the specific terms should be introduced. Students should be encouraged to use their own personal vocabulary as they develop their understandings and to then demonstrate the new terminology as they summarize their discoveries and learnings for themselves and for others in the classroom.

In helping students develop their working mathematical language, it is also important for the teacher to recognize that for many students, including First Nations and Métis, that just because a student doesn’t recognize a specific term or procedure, the student may in fact have a deep understanding of the overall mathematical topic. Many perceived learning difficulties in mathematics are the result of students’ cultural and personal ways of knowing not being connected to the mathematical language.

In addition, the English language often allows for multiple interpretations of the same sentence, depending upon where the emphasis is placed. For example, consider the sentence “The shooting of the hunters was terrible” (Paulos, 1980, p. 65). Were the hunters that bad of a shot, was it terrible that the hunters got shot, was it terrible that they were shooting, or is this all about the photographs that were taken of the hunters? It is important that students be engaged in dialogue through which they explore possible meanings and interpretations of mathematical statements and problems.

First Nations and Métis Learners and Mathematics

It is important for teachers to realize that First Nations and Métis students, like all students, come to mathematics classes with a wealth of mathematical understandings. Within these mathematics classes, some First Nations and Métis students may develop a negative sense of their ability in mathematics and, in turn, do poorly on mathematics assessments. Such students may become alienated from mathematics because it

is not taught to their schema, cultural and environmental content, or real life experiences. A first step in actualization of mathematics from First Nations and Métis perspectives is to empower teachers to understand that mathematics is not acultural. As a result, teachers then realize that the traditional ways of teaching the mathematics are also culturally-biased. These understandings will support the teacher in developing First Nations and Metis students' personal mathematical understandings and mathematical self-confidence and ability through a more holistic and constructivist approach to learning. Teachers need to consider factors that impact the success of First Nations and Métis students in mathematics: cultural contexts and pedagogy.

It is important for teachers to recognize the influence of cultural contexts on mathematical learning. Educators need to be sensitive to the cultures of others, as well as to how their own cultural background influences their current perspective and practice. Mathematics instruction focuses on the individual parts of the whole understanding and, as a result, the contexts presented tend to be compartmentalized and treated discretely. This focus on parts may be challenging for students who rely on whole contexts to support understanding.

Mathematical ideas are valued, viewed, contextualized, and expressed differently by cultures and communities. Translation of these mathematical ideas between cultural groups cannot be assumed to be a direct link. Consider, for example, the concept of "equal", which is a key understanding in this curriculum. The Western understanding of "equal" is 'the same'. In many First Nations and Métis communities, however, "equal" is understood as meaning 'for the good of the community'. Teachers need to support students in uncovering these differences in ways of knowing and understanding within the mathematic classroom. Various ways of knowing need to be celebrated to support the learning of all students.

Along with an awareness of students' cultural context, pedagogical practices also influence the success of First Nations and Métis students in the mathematics classroom. Mathematical learning opportunities need to be holistic, occurring within social and cultural interactions through dialogue, language, and the negotiation of meanings. Constructivism, ethnomathematics, and teaching through an inquiry approach are supportive of a holistic perspective to learning. Constructivism, inquiry learning, and ethnomathematics allow students to enter the learning process according to their ways of knowing, prior knowledge, and learning styles. Ethnomathematics also shows the relationship between mathematics and cultural anthropology. It is used to translate earlier forms of thinking into modern-day understandings. Individually, and as a class, teachers and students need to explore the big ideas that are foundational to this curriculum and investigate how those ideas relate to them personally and as a learning community. Mathematics learned within contexts that focus on the day-to-day activities found in students' communities support learning by providing a holistic focus. Mathematics needs to be taught using the expertise of elders and the local environment as educational resources. The variety of interactions that occur among students, teachers, and the community strengthen the learning experiences for all.

Critiquing Statements

One way to assess students' depth of understanding of an outcome is to have the students critique a general statement which, on first reading, may seem to be true or false. By having students critique such statements, the teacher is able to identify strengths and deficiencies in their understanding. Some indicators in this curriculum are examples of statements that students can analyze for accuracy. For example, for outcome P9.3, one of the indicators reads:

Critique the statement: “For any linear equality, there are two related linear inequalities”.

The purpose of this indicator is for teachers to assess the depth of understanding students have about the relationship between equality and inequality in a mathematical, and more specifically variable, context. Although it may be true that one quantity can always be described as being equal to, less than, or greater than another quantity, the same is not true of variable expressions. In such cases, two additional possibilities exist in the relationship, that of being less than or equal to and greater than or equal to. Students often find this conceptually difficult if they have not grasped the relationship, and also the fundamental difference between quantities and variable expressions. Asking students to critique statements like the one given above will give teachers insight into the students’ understandings, and also provide guidance regarding further experiences that the student may need to have.

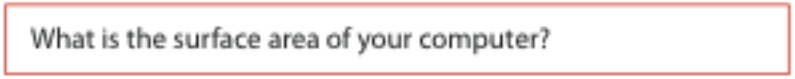
Critiquing statements is an effective way to assess students individually or as a small or large group. When engaged as a group, the discussion and strategies that emerge not only inform the teacher, but also engage all of the students in a deeper understanding of the topic.

The Concrete to Abstract Continuum

It is important that, in learning mathematics, students be allowed to explore and develop understandings by moving along a concrete to abstract continuum. As understanding develops, this movement along the continuum is not necessarily linear. Students may at one point be working abstractly but when a new idea or context arises, they need to return to a concrete starting point. Therefore, the teacher must be prepared to engage students at different points along the continuum.

In addition, what is concrete and what is abstract is not always obvious and can vary according to the thinking processes of the individual. For example, when considering a problem about the total number of pencils, some students might find it more concrete to use pictures of pencils as a means of representing the situation. Other students might find coins more concrete because they directly associate money with the purchasing or having of a pencil.

As well, teachers need to be aware that different aspects of a task might involve different levels of concreteness or abstractness. Consider the following situational question involving surface area:



Depending upon how the question is expected to be solved (or if there is any specific expectation), this question can be approached abstractly (using symbolic number statements), concretely (e.g., using manipulatives, pictures), or both.

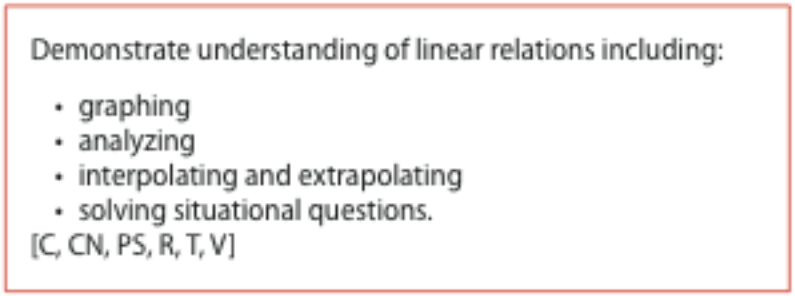
Models and Connections

New mathematics is continuously developed by creating new models as well as combining and expanding existing models. Although the final products of mathematics are most frequently represented by symbolic models, their meaning and purpose is often found in the concrete, physical, pictorial, and oral models and the connections between them.

To develop a deep and meaningful understanding of mathematical concepts, students need to represent their ideas and strategies using a variety of models (concrete, physical, pictorial, oral, and symbolic). In addition, students need to make connections between the different representations. These connections are made by having the students try to move from one type of representation to another (how could you write what you've done here using mathematical symbols?) or by having students compare their representations with others in the class.

In making these connections, students should be asked to reflect upon the mathematical ideas and concepts that are being used in their new models (e.g., I know that addition means to put things together into a group, so I'm going to move the two sets of alge-tiles together to determine the sum of the polynomials).

Making connections also involves looking for patterns. For example, in outcome P9.1:


Demonstrate understanding of linear relations including:

- graphing
- analyzing
- interpolating and extrapolating
- solving situational questions.

[C, CN, PS, R, T, V]

the students' exploration and recognition of patterns and relationships between graphs and equations of linear relations is key to the development of a deep understanding of linear relations. Students need to build strong connections between the characteristics and properties of the graphs and the characteristics of the equations.

Role of Homework

The role of homework in teaching for deep understanding is important. Students should be given unique problems and tasks that help students to consolidate new learnings with prior knowledge, explore possible solutions, and apply learnings to new situations. Although drill and practice does serve a purpose in learning for deep understanding, the amount and timing of the drill will vary among different learners. In addition, when used as homework, drill and practice frequently serves to cause frustration, misconceptions, and boredom to arise in students.

As an example of the type or style of homework that can be used to help students develop deep understanding of Grade 9 Mathematics, consider outcome SP9.3:

> SP9.3 Demonstrate an understanding of the role of probability in society.
>
> [C, CN, R, T]

As a homework task, students might be asked to collect examples from their homes, interests, and personal lives that involve or include statements of probability. From there, those examples could be shared in class and would provide the foundation for the students' learning about the impact of probability theory on their everyday lives. Some of the questions that students might discuss related to their examples include:

- Would you make the same decision as the person/people in your example? Why or why not?
- Are there factors other than the known probability that you would consider before making such a decision?
- Is an 80% chance of an event happening equally influential in different contexts?

By first engaging the students in contexts that are personally relevant, students can explore and bring to light different factors to consider when making a decision regarding known probability. From these personal contexts, students can expand into community and socially relevant contexts and bring a better understanding of the role of emotions, values, and probability in decision making.

Ongoing Feedback and Reflection

Ongoing feedback and reflection, both for students and teachers, are crucial in classrooms when learning for deep understanding. Deep understanding requires that both the teacher and students need to be aware of their own thinking as well as the thinking of others.

Feedback from peers and the teacher helps students rethink and solidify their understandings. Feedback from students to the teacher gives much needed information in the teacher's planning for further and future learnings.

Self-reflection, both shared and private, is foundational to students developing a deep understanding of mathematics. Through reflection tasks, students and teachers come to know what it is that students do and do not know. It is through such reflections that not only can a teacher make better informed instructional decisions, but also that a student can set personal goals and make plans to reach those goals.

Teaching for Deep Understanding

For deep understanding, it is vital that students learn by constructing knowledge, with very few ideas being relayed directly by the teacher. As an example, the addition sign (+) is something which the teacher must introduce and ensure that students know. It is the symbol used to show the combination or addition of two quantities. The process of adding, however, and the development of addition and subtraction facts should be discovered through the students' investigation of patterns, relationships, abstractions, and generalizations.

It is important for teachers to analyze the outcomes to identify what students need to know, understand, and be able to do. Teachers also need to consider opportunities they can provide for students to explain, apply, and transfer understanding to new situations. This reflection supports professional decision making and planning effective strategies to promote students' deeper understanding of mathematical ideas.

It is important that a mathematics learning environment include effective interplay of:

- reflection
- exploration of patterns and relationships
- sharing of ideas and problems
- consideration of different perspectives
- decision making
- generalizing
- verifying and proving
- modeling and representing.

Mathematics is learned when students are engaged in strategic play with mathematical concepts and differing perspectives. When students learn mathematics by being told what to do, how to do it, and when to do it, they cannot make the strong connections necessary for learning to be meaningful, easily accessible, and transferable. The learning environment must be respectful of individuals and groups, fostering discussion and self-reflection, the asking of questions, the seeking of multiple answers, and the construction of meaning.

Inquiry

Inquiry learning provides students with opportunities to build knowledge, abilities, and inquiring habits of mind that lead to deeper understanding of their world and human experience. The inquiry process focuses on the development of compelling questions, formulated by teachers and students, to motivate and guide inquiries into topics, problems, and issues related to curriculum content and outcomes.

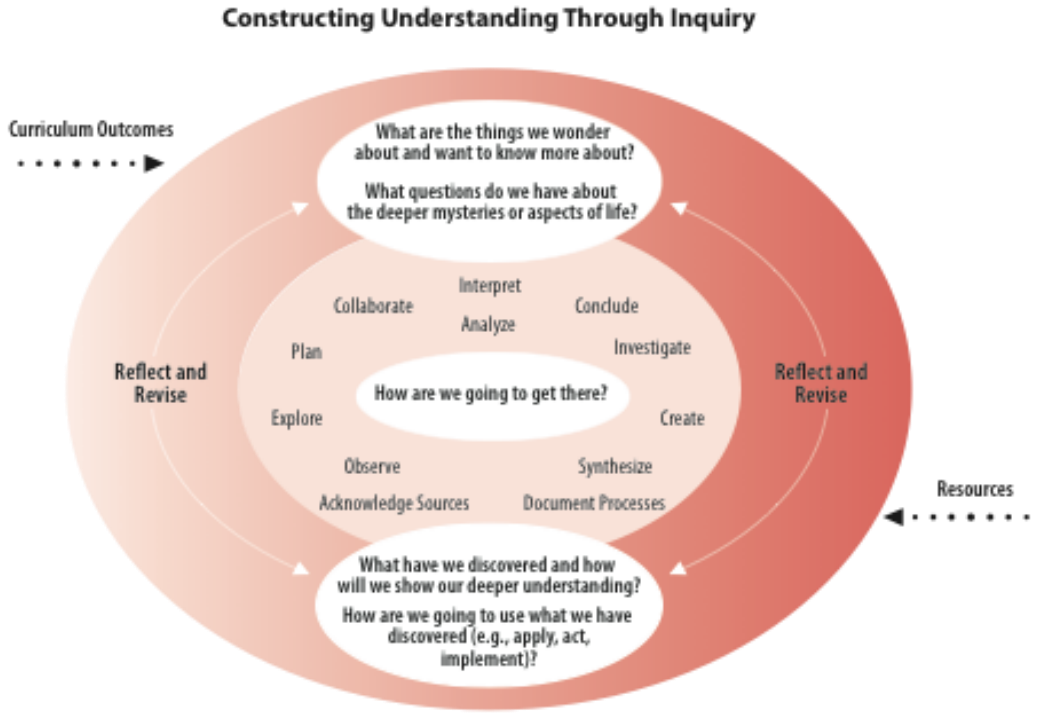
Inquiry is more than a simple instructional method. It is a philosophical approach to teaching and learning, grounded in constructivist research and methods, which engages students in investigations that lead to disciplinary and transdisciplinary understanding.

Inquiry builds on students' inherent sense of curiosity and wonder, drawing on their diverse backgrounds, interests, and experiences. The process provides opportunities for students to become active participants in a collaborative search for meaning and understanding. Students who are engaged in inquiry:

- construct deep knowledge and deep understanding rather than passively receiving it
- are directly involved and engaged in the discovery of new knowledge
- encounter alternative perspectives and conflicting ideas that transform prior knowledge and experience into deep understanding
- transfer new knowledge and skills to new circumstances
- take ownership and responsibility for their ongoing learning and mastery of curriculum content and skills.

(Adapted from Kuhlthau & Todd, 2008, p. 1)

Inquiry learning is not a step-by-step process, but rather a cyclical process, with various phases of the process being revisited and rethought as a result of students' discoveries, insights, and construction of new knowledge. The following graphic shows the cyclical inquiry process.



Inquiry prompts and motivates students to investigate topics within meaningful contexts. The inquiry process is not linear or lock-step, but is flexible and recursive. Experienced inquirers move back and forth through the cyclical process as new questions arise and as students become more comfortable with the process.

Well-formulated inquiry questions are broad in scope and rich in possibilities. They encourage students to explore, gather information, plan, analyze, interpret, synthesize, problem solve, take risks, create, conclude, document, reflect on learning, and develop new questions for further inquiry.

In mathematics, inquiry encompasses problem solving. Problem solving includes processes to get from what is known to discover what is unknown. When teachers show students how to solve a problem and then assign additional problems that are similar, the students are not problem solving but practising. Both are necessary in mathematics, but one should not be confused with the other. If the path for getting to the end situation has already been determined, it is no longer problem solving. Students too must understand this difference.

Creating Questions for Inquiry in Mathematics

Teachers and students can begin their inquiry at one or more curriculum entry points; however, the process may evolve into trans-disciplinary integrated learning opportunities, as reflective of the holistic nature of our lives and interdependent global environment. It is essential to develop questions that are evoked by students' interests and have potential for rich and deep learning. Compelling questions are used to initiate and guide the inquiry and give students direction for discovering deep understandings about a topic or issue under study.

The process of constructing inquiry questions can help students to grasp the important disciplinary or trans-disciplinary ideas that are situated at the core of a particular curricular focus or context. These broad questions will lead to more specific questions that can provide a framework, purpose, and direction for the learning activities in a lesson, or series of lessons, and help students connect what they are learning to their experiences and life beyond school.

Effective questions in mathematics are the key to initiating and guiding students' investigations, critical thinking, problem solving, and reflection on their own learning. Questions such as:

- "When would you want to add two numbers less than 100?"
- "How do you know you have an answer?"
- "Will this work with every number? Every similar situation?"
- "How does your representation compare to that of your partner?"

are examples of questions that will move students' inquiry towards deeper understanding. Effective questioning is essential for teaching and student learning, and should be an integral part of planning in mathematics. Questioning should also be used to encourage students to reflect on the inquiry process and the documentation and assessment of their own learning.

Questions should invite students to explore mathematical concepts within a variety of contexts and for a variety of purposes. When questioning students, teachers should choose questions that:

- *help students make sense of the mathematics.*
- *are open-ended, whether in answer or approach. There may be multiple answers or multiple approaches.*
- *empower students to unravel their misconceptions.*
- *not only require the application of facts and procedures but encourage students to make connections and generalizations.*
- *are accessible to all students in their language and offer an entry point for all students.*
- *lead students to wonder more about a topic and to perhaps construct new questions themselves as they investigate this newly found interest.*

(Schuster & Canavan Anderson, 2005, p. 3)

Reflection and Documentation of Inquiry

An important part of any inquiry process is student reflection on their learning and the documentation needed to assess the learning and make it visible. Student documentation of the inquiry process in mathematics may take the form of reflective journals, notes, drafts, models, and works of art, photographs, or video footage. This documentation should illustrate the students' strategies and thinking processes that led to new insights and conclusions. Inquiry-based documentation can be a source of rich assessment materials through which teachers can gain a more in-depth look into their students' mathematical understandings.

It is important that students are required to engage in the communication and representation of their progress within a mathematical inquiry. A wide variety of forms of communication and representation should be encouraged and, most importantly, have links made between them. In this way, student inquiry into mathematical concepts and contexts can develop and strengthen student understanding.

Summary of Math 18 and 28 Outcomes and Indicators

NUMBER

Math 18	Math 28
N18.1 Demonstrate an understanding of whole numbers to 1 000. a. Represent numbers to 1 000. b. Describe numbers to 1 000. c. Compare two numbers up to 1 000. d. Order three or more numbers up to three digits. e. Hear a number up to three digits and write down the numeral. f. Skip count forward by 2's, 5's, 10's, and 100's. g. Skip count backward by 2's, 5's, 10's, and 100's.	**N28.1 Demonstrate an understanding of whole numbers to 1 000 000.** a. Represent numbers to 1 000 000. b. Describe numbers to 1 000 000. c. Compare two numbers up to 1 000 000. d. Order three or more numbers with up to six digits. e. Hear a number up to six digits and write down the numeral. f. Skip count forward by 2's, 3's, 4's, 5's, 10's, 25's, and 100's using any starting point. g. Skip count backward by 2's, 3's, 4's, 5's, 10's, 25's, and 100's using any starting point.
N18.2 Represent, compare and describe whole numbers to 1 000 within the contexts of place value and the base ten system. a. Write and say the numeral for a quantity using proper spacing. b. Identify the position of a digit in a given numeral. c. Pose and solve problems that explore the quantity of whole numbers. d. Express a number in expanded notation.	
	N28.2 Develop and apply personal strategies for estimation and computation. a. Front-end rounding. b. Compensation. c. Compatible numbers.

N18.3 Demonstrate an understanding of addition of whole numbers (up to 3 digits). a. Use personal strategies (including hundreds chart, ruler, manipulatives, calculator, etc.) for adding whole numbers. b. Estimate sums. c. Solve problems involving addition. d. Create problems involving addition. e. Relate subtraction to addition.	**N28.3 Extend understanding of addition, subtraction, multiplication, and division of whole numbers by solving single variable problems with whole numbers.** a. Identify aspects from daily life that could be represented by a variable (e.g., temperature, cost of an iPad, performance of a pitcher, etc.). b. Describe a situation for which a given equation could apply and identify what the variable would be. c. Solve single variable equations. d. Explain the strategies used to solve an equation. e. Verify the results of an equation.
N18.4 Demonstrate an understanding of subtraction of whole numbers (up to 3 digits). a. Use personal strategies (including hundreds chart, ruler, manipulatives, calculator, etc.) for subtracting whole numbers. b. Estimate differences. c. Solve problems involving subtraction. d. Create problems involving subtraction.	
N18.5 Demonstrate an understanding of multiplication of whole numbers (up to 3 digits x 1 digit multipliers). a. Use personal strategies (including a calculator, manipulatives, multiplication chart, etc.) to multiply whole numbers. b. Use arrays to represent multiplication. c. Estimate products. d. Solve problems involving multiplication. e. Create problems involving multiplication. f. Relate multiplication to division.	

N18.6 Demonstrate an understanding of division of whole numbers (up to 3 digits divided by 1 digit divisors). a. Use personal strategies (including a calculator, manipulatives, multiplication chart, etc.) to divide whole numbers. b. Estimate quotients. c. Solve problems involving division. d. Create problems involving division.	
N18.7 Demonstrate an understanding of fractions less than or equal to one. a. Represent a fraction using concrete materials. b. Name and record fractions for the parts of a whole or a set. c. Compare and order fractions with similar denominators. d. Create sets of equivalent fractions. e. Provide examples of fractions used in daily life. f. Provide examples of when two identical fractions may not represent the same quantity.	**N28.4 Extend understanding of fractions to improper fractions and mixed numerals.** a. Identity equivalent fractions. b. Convert a mixed number to a fraction and a fraction to a mixed number. c. Add and subtract proper and improper fractions with like denominators. d. Provide examples of improper fractions and mixed numerals in daily life.
N18.8 Demonstrate an understanding of decimals to the hundredths. a. Describe and represent decimal numbers to the hundredths. b. Add and subtract decimal numbers to the hundredths. c. Estimate sums and differences of decimal numbers to the hundredths. d. Solve problems with decimals to the hundredths. e. Relate decimal numbers to fractions.	**N28.5 Extend understanding of decimals to the hundredths.** a. Multiply and divide decimal numbers to the hundredths (with one-digit whole number multipliers or divisors). b. Estimate products and quotients of decimal numbers to the hundredths. c. Solve problems with decimals to the hundredths. d. Express a fraction as a decimal and a decimal as a fraction.
	N28.6 Demonstrate an understanding of percent. a. Describe examples of percents in daily life. b. Describe situations in which 0% or 100% may be used. c. Write percent as a decimal or fraction. d. Write a decimal as a percent. e. Write a fraction as a percent.

	N28.7 Operate a calculator accurately. a. Accurately calculate an operation involving the addition, subtraction, multiplication, or division of whole numbers. b. Accurately calculate an operation involving the addition, subtraction, multiplication, or division of fractions and/or mixed numbers. c. Accurately calculate an operation involving the addition, subtraction, multiplication, or division of decimals. d. Find the percent of a number using a calculator. e. Calculate sales tax. f. Calculate discounts.
PATTERNS AND RELATIONS	
Math 18	**Math 28**
PR18.1 Demonstrate an understanding of patterns and relations in tables, charts, diagrams and graphs. a. Identify and describe patterns on a multiplication chart. b. Describe and extend patterns found in a table or chart. c. Identify the sorting rule for a Venn diagram. d. Describe the relationship shown in a given Venn diagram where the circles intersect, when one circle is contained in the other, and when the circles are separate. e. Determine where new data should be placed in a Venn diagram. f. Compare graphs in which the same data has been displayed using one-to-one correspondences.	**PR28.1 Extend understanding of patterns and relations in tables, charts, diagrams and graphs.** a. Create a chart or table based on provided information. b. Solve problems using charts, tables, or diagrams to identify mathematical relationships. c. Compare graphs in which the same data has been displayed using one-to-one correspondence and many-to-one correspondence and explain how they are similar and different. d. Compare graphs in which different correspondences are used and explain why the correspondences may have been used.
Shape and Space	
Math 18	**Math 28**

SS18.1 Demonstrate an understanding of time. a. Use standard and non-standard units of measure to mark the passage of time. b. Understand the concept of time using years, months, weeks, days, hours, and minutes. c. Describe activities related to daily life that involve the measurement of time. d. Identify today's date, day of the week, month of the year, and year. e. Locate a specific date on the calendar and identify strategy used to locate date. f. Order events according to time. g. Read and record time using 12-hour digital and analog clocks.	**SS28.1 Extend understanding of time.** a. Express time orally on both a 12-hour and 24-hour analog and digital clock. b. Write down time shown on both a 12-hour and 24-hour analog and digital clock. c. Express time as "minutes to" or "minutes after" the hour. d. Explain the meaning of AM and PM and provide examples of activities that occur in the AM and PM. e. Read and record calendar dates in a variety of formats.
SS18.2 Demonstrate an understanding of symmetry. a. Sort 2-D shapes into symmetrical and non-symmetrical categories. b. Identify lines of symmetry and explain why each shape is symmetrical. c. Determine whether or not a 2-D shape is symmetrical by folding or super-imposing.	
SS18.3 Demonstrate an understanding of regular and irregular 2-D shapes. a. Identify examples of 2-D shapes in the classroom, home, and community. b. Identify circles, squares, triangles, rectangles, semicircles, hexagons, octagons, pentagons, trapezoids, parallelograms, and rhombuses. c. Create and describe a concrete representation of a personally relevant 2-D shape. d. Sort quadrilaterals into rectangles, squares, trapezoids, parallelograms, and rhombuses.	**SS28.2 Extend understanding of regular and irregular 2-D shapes.** a. Calculate the perimeter of a variety of 2-D shapes. b. Calculate the area of a variety of 2-D shapes using the area formula.

SS18.4 Demonstrate an understanding of 3-D objects. a. Identify examples of 3-D objects in the classroom, home, and community. b. Identify pyramids, cubes, spheres, cones, and cylinders. c. Create and describe a concrete representation of a personally relevant 3-D object. d. Sort 3-D objects according to an attribute and describe the rule.	**SS28.3 Demonstrate an understanding of rectangular and triangular prisms.** a. Sort a set of rectangular and triangular prisms using the shape of the base. b. Identify examples of rectangular and triangular prisms in the environment. c. Construct rectangular and triangular prisms.
SS18.5 Demonstrate an understanding of mass (g and kg). a. Describe the relationship between g and kg. b. Estimate mass using g and kg. c. Measure and record mass using g and kg.	
SS18.6 Demonstrate an understanding of linear measurement (mm, cm, and m). a. Describe the relationship between mm, cm, and m. b. Estimate length and perimeter. c. Measure and record length, width, height, and perimeter. d. Calculate the perimeter of various shapes. e. Demonstrate an understanding of circles, including circumference. f. Demonstrate an understanding of area of a square, rectangle, triangle, and circle.	**SS28.4 Extend understanding of linear measurement (mm, cm, and m).** a. Sketch a line segment of an estimated length and describe the strategy used. b. Draw a 2-D shape given the measurements and/or perimeter of the shape.
SS18.7 Demonstrate an understanding of capacity (ml and l). a. Describe the relationship between ml and l. b. Estimate capacity using ml and l. c. Measure and record capacity using ml and l.	**SS28.5 Extend understanding of capacity to daily life.** a. Identify the unit of measure in a recipe. b. Measure the correct amount of ingredients with measuring utensils.
	SS28.6 Demonstrate an understanding of angles. a. Estimate the measure. b. Determine the angle measure in degrees. c. Draw angles.

	SS28.7 Demonstrate an understanding of temperature and how it relates to daily life. a. Accurately read a thermometer in degrees Celsius. b. Determine how temperature relates to daily life. c. Apply temperature to daily decisions (cooking, dressing, etc.).
Statistics and Probability	
Math 18	**Math 28**
SP18.1 Demonstrate an understanding of graphs. a. Formulate a question relevant to one's self, family, or community that can be answered by gathering information from people. b. Pose questions related to gathered data and explain how data can be used to answer those questions. c. Create a graph to display collected data.	**SP28.1 Extend an understanding of graphs.** a. Find examples of graphs in which a many-to-one correspondence is used in print and electronic media, such as newspapers, magazines, and the internet, and describe the correspondence used. b. Create and label (with categories, title, and legend) a pictograph to display a set of data using a many-to-one correspondence, and justify the choice of correspondence used. c. Create and label (with axes and title) a bar graph to display a set of data using a many-to-one correspondence, and justify the choice of correspondence used. d. Answer a question using a graph in which data is displayed using a many-to-one correspondence.
SP18.2 Demonstrate an understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots. a. Observe and describe situations relevant to self, family, or community in which a particular type of data recording or organizational strategy might be used, including tally marks, charts, lists, and knots on a sash. b. Create a line plot from a pictograph. c. Answer questions related to the data presented in a bar graph or line plot. d. Collect and represent data using bar graphs or line plots.	**SP28.2 Extend understanding of data analysis.** a. Create graphs from given data. b. Collect data through questionnaires. c. Collect data through experiments. d. Draw conclusions from data collected. e. Report conclusions.

Consumer Math	
Math 18	**Math 28**
CM18.1 Demonstrate an understanding of coins and bills up to $100.00. a. Identify names and values of coins and bills up to $100.00. b. Count bills and coins up to $100.00. c. Make change by using subtraction and counting back.	
	CM28.1 Demonstrate an understanding of income. a. Calculate daily and weekly wages involving regular pay. b. Determine the difference between gross and net pay by calculating deductions using CPP, income tax, and EI charts.
CM18.2 Apply understanding of money. a. Calculate the unit price of items sold in multiples. b. Compare prices between products based on equivalent number and sizes.	**CM28.2 Demonstrate an understanding of budget.** a. Define and explain the purpose of a budget. b. Determine fixed and variable expenses for the month. c. Plan a budget based on percentages allotted to various categories as suggested by financial institutions. d. Stay within a budget and adjust a budget as necessary. e. Estimate the cost of groceries and decide if you can make the purchase. f. Calculate the cost of a purchase.
	CM28.3 Demonstrate an understanding of credit. a. Define credit and determine its appropriate use. b. Determine why a good credit rating is important. c. Describe how to get a good credit rating. d. Describe how to use credit wisely. e. Find the monthly interest charges and service charges on unpaid credit balance.

	CM28.4 Demonstrate an understanding of banking. a. Know the definitions for banking terms (interest, deposit overdraft, withdrawal, service charges, safety deposit box, travelers' cheques, balance, debit, credit, NSF cheques, bonds, bounced cheques). b. Know the difference between a chequing account and a savings account. c. Write a cheque and know how to cash it. d. Know how to use an ATM machine and a debit card. e. Calculate simple interest with the use of a calculator or chart.
	CM28.5 Demonstrate a basic understanding of loans. a. Identify reasons for getting a loan. b. Complete a loan application. c. Determine the cost of the loan and how it fits within a budget.

Outcomes and Indicators

Number

N18.1 Demonstrate an understanding of whole numbers to 1 000.

a. Represent numbers to 1 000.
b. Describe numbers to 1 000.
c. Compare two numbers up to 1 000.
d. Order three or more numbers up to three digits.
e. Hear a number up to three digits and write down the numeral.
f. Skip count forward by 2's, 5's, 10's, and 100's.
g. Skip count backward by 2's, 5's, 10's, and 100's.

N18.2 Represent, compare and describe whole numbers to 1 000 within the contexts of place value and the base ten system.

a. Write and say the numeral for a quantity using proper spacing.
b. Identify the position of a digit in a given numeral.
c. Pose and solve problems that explore the quantity of whole numbers.

d. Express a number in expanded notation.

N18.3 Demonstrate an understanding of addition of whole numbers (up to 3 digits).

a. Use personal strategies (including hundreds chart, ruler, manipulatives, calculator, etc.) for adding whole numbers.
b. Estimate sums.
c. Solve problems involving addition.
d. Create problems involving addition.
e. Relating subtraction to addition.

N18.4 Demonstrate an understanding of subtraction of whole numbers (up to 3 digits).

a. Use personal strategies (including hundreds chart, ruler, manipulatives, calculator, etc.) for subtracting whole numbers.
b. Estimate differences.
c. Solve problems involving subtraction.
d. Create problems involving subtraction.

N18.5 Demonstrate an understanding of multiplication of whole numbers (up to 3 digits x 1 digit multipliers).

a. Use personal strategies (including a calculator, manipulatives, multiplication chart, etc.) to multiply whole numbers.
b. Use arrays to represent multiplication.
c. Estimate products.
d. Solve problems involving multiplication.
e. Create problems involving multiplication.
f. Relate multiplication to division.

N18.6 Demonstrate an understanding of division of whole numbers (up to 3 digits divided by 1 digit divisors).

a. Use personal strategies (including a calculator, manipulatives, multiplication chart, etc.) to divide whole numbers.
b. Estimate quotients.
c. Solve problems involving division.
d. Create problems involving division.

N18.7 Demonstrate an understanding of fractions less than or equal to one.

a. Represent a fraction using concrete materials.

b. Name and record fractions for the parts of a whole or a set.
c. Compare and order fractions with similar denominators.
d. Create sets of equivalent fractions.
e. Provide examples of fractions used in daily life.
f. Provide examples of when two identical fractions may not represent the same quantity.

N18.8 Demonstrate an understanding of decimals to the hundredths.

a. Describe and represent decimal numbers to the hundredths.
b. Add and subtract decimal numbers to the hundredths.
c. Estimate sums and differences of decimal numbers to the hundredths.
d. Solve problems with decimals to the hundredths.
e. Relate decimal numbers to fractions.

Patterns and Relations

PR18.1 Demonstrate an understanding of patterns and relations in tables, charts, diagrams and graphs.

a. Identify and describe patterns on a multiplication chart.
b. Describe and extend patterns found in a table or chart.
c. Identify the sorting rule for a Venn diagram.
d. Describe the relationship shown in a given Venn diagram where the circles intersect, when one circle is contained in the other, and when the circles are separate.
e. Determine where new data should be placed in a Venn diagram.
f. Compare graphs in which the same data has been displayed using one-to-one correspondences.

Shape and Space

SS18.1 Demonstrate an understanding of time.

a. Use standard and non-standard units of measure to mark the passage of time.
b. Understand the concept of time using years, months, weeks, days, hours, and minutes.
c. Describe activities related to daily life that involve the measurement of time.
d. Identify today's date, day of the week, month of the year, and year.

e. Locate a specific date on the calendar and identify strategy used to locate date.
f. Order events according to time.
g. Read and record time using 12-hour digital and analog clocks.

SS18.2 Demonstrate an understanding of symmetry.

a. Sort 2-D shapes into symmetrical and non-symmetrical.
b. Identify lines of symmetry and explain why each shape is symmetrical.
c. Determine whether a 2-D shape is symmetrical by folding or superimposing.

SS18.3 Demonstrate an understanding of regular and irregular 2-D shapes.

a. Identify examples of 2-D shapes in the classroom, home, and community.
b. Identify circles, squares, triangles, rectangles, semicircles, hexagons, octagons, pentagons, trapezoids, parallelograms, and rhombuses.
c. Create and describe a concrete representation of a personally relevant 2-D shape.
d. Sort quadrilaterals into rectangles, squares, trapezoids, parallelograms, and rhombuses.

SS18.4 Demonstrate an understanding of 3-D objects.

a. Identify examples of 3-D objects in the classroom, home, and community.
b. Identify pyramids, cubes, spheres, cones, and cylinders.
c. Create and describe a concrete representation of a personally relevant 3-D object.
d. Sort 3-D objects according to an attribute and describe the rule.

SS18.5 Demonstrate an understanding of mass (g and kg).

a. Describe the relationship between g and kg.
b. Estimate mass using g and kg.
c. Measure and record mass using g and kg.

SS18.6 Demonstrate an understanding of linear measurement (mm, cm, and m).

a. Describe the relationship between mm, cm, and m.
b. Estimate length and perimeter.
c. Measure and record length, width, height, and perimeter.

d. Calculate the perimeter of various shapes.
e. Demonstrate an understanding of circles, including circumference.
f. Demonstrate an understanding of area of a square, rectangle, triangle, and circle.

SS18.7 Demonstrate an understanding of capacity (ml and l).

a. Describe the relationship between ml and l.
b. Estimate capacity using ml and l.
c. Measure and record capacity using ml and l.

Statistics and Probability

SP18.1 Demonstrate an understanding of graphs.

a. Formulate a question relevant to one's self, family, or community that can be answered by gathering information from people.
b. Pose questions related to gathered data and explain how data can be used to answer those questions.
c. Create a graph to display collected data.

SP18.2 Demonstrate an understanding of first-hand data using tally marks, chars, lists, bar graphs, and line plots.

a. Observe and describe situations relevant to self, family, or community in which a particular type of data recording or organizing strategy might be used, including tally marks, charts, lists, and knots on a sash.
b. Create a line plot from a pictograph.
c. Answer questions related to the data presented in a bar graph or line plot.
d. Collect and represent data using bar graphs or line plots.

Consumer Math

CM18.1 Demonstrate an understanding of coins and bills up to $100.00.

a. Identify names and values of coins and bills up to $100.00.
b. Count bills and coins up to $100.00.
c. Make change by using subtraction and counting back.

CM18.2 Apply understanding of money.

a. Calculate the unit price of items sold in multiples.

b. Compare prices between products based on equivalent number and sizes.

Number

UbD Planning Document

Good Spirit School Division UbD Unit Plan		
Teacher:	**Subject: Math**	**Grade: Ten – Level 18**
Unit Title: Number		
Context (ELA only):	**Type of Unit (ELA only):**	
Time Frame:		
STAGE ONE: IDENTIFY THE DESIRED RESULTS		
Outcomes Addressed in the Unit **SK curriculum outcomes can be copied and pasted, focuses highlighted.**		
N18.1 Demonstrate an understanding of whole numbers to 1 000. N18.2 Represent, compare, and describe whole numbers to 1 000 within the contexts of place value and the base ten system. N18.3 Demonstrate an understanding of addition of whole numbers (up to 3 digits). N18.4 Demonstrate an understanding of subtraction of whole numbers (up to 3 digits). N18.5 Demonstrate an understanding of multiplication of whole numbers (up to 3 digits x 1 digit multipliers). N18.6 Demonstrate an understanding of division of whole numbers (up to 3 digits divided by 1 digit divisors). N18.7 Demonstrate an understanding of fractions less than or equal to one. N18.8 Demonstrate an understanding of decimals to the hundredths.		
Big Ideas/Enduring Understandings **What do you want students to understand and be able to use several years from now?** **What are the BIG ideas?**	**Essential Questions** **Open-ended questions that stimulate thought and inquiry linked to the content of the enduring understandings.**	

- People in different cultures represent numbers in different ways. - We use fractions and decimals in real life. - Addition, subtraction, multiplication, and division are related operations that involve taking numbers apart and combining numbers.	- What are some ways that people in different cultures represent numbers? - Why is place value important? - How are addition and subtraction related? - How are multiplication and division related? - When do we use fractions in real life? - How can the same fraction mean different things?
Knowledge and Skills (Students will know and do…) **What key knowledge and skills will students acquire as a result of this unit?** **(These may be indicators from the curriculum)**	
Knowledge (Students will know…) **What key knowledge will students acquire as a result of this unit?**	**Skills (Students will know how to…)** **What key skills will students acquire as a result of this unit?**
- what a whole number is. - how to read numbers. - the concept of greater than and less than. - that numbers can be represented in different ways. - the value of a numeral depends on its place in the number. - skip counting follows a pattern. - basic math facts. - that addition and subtraction are related. - addition results in larger quantities and subtraction in smaller quantities. - place value is important in understanding addition and subtraction. - how multiplication and division are related to addition and subtraction. - the relationship between an array and a multiplication question. - that multiplication and division are related. - what a fraction is. - the role of the numerator and the denominator. - when fractions are used. - why we use fractions.	- say and write whole numbers. - how to add, subtract, multiply, and divide whole numbers - represent whole numbers concretely, pictorially, in writing, and symbolically. - explain the meaning of each digit in a 3-digit number - show different decompositions of a given number in different ways - write and read real-life numbers up to 1 000. - create and solve real-life problems. - explain personal strategies for solving problems. - represent an addition, subtraction, multiplication, and division statement in a variety of ways. - skip count forwards and backwards. - order fractions with common numerators and with common denominators. - represent fractions and decimals in a variety of ways. - use fractions in real-life situations.
STAGE TWO: DESIGN ASSESSMENT EVIDENCE	
Assessment Evidence	

<table>
<tr><th colspan="2"><u>Summative Assessments/Performance Tasks</u>
Assessments of what students know and can do aligned to the outcomes. They are a snapshot in time used for reporting and evaluating.</th></tr>
<tr><td>Outcomes/Objectives</td><td>Co-construct criteria and/or rubrics with students whenever possible. Examples of Summative Assessments/Performance Tasks include:</td></tr>
<tr><td rowspan="3">N18.1, N18.2, N18.3, N18.4, N18.5, N18.6, N18.7, N18.8</td><td>Compose a Test - Ask students to compose a test with an answer key about what they have learned during their study of the number outcomes. Interview students or have them respond in writing about their reasons for choosing the questions they included on their test.</td></tr>
<tr><td>Examples/Non-Examples - To find out what students have learned about different concepts and skills, ask students to provide examples and non-examples. For example, students can provide examples and non-examples related to place value, fractions, multiplication, etc.</td></tr>
<tr><td>How-To Book - Provide students with a list of concepts and/or skills that they worked on during the number unit. Ask students to create a “how-to” book that focuses on these concepts/skills. For example, how to solve a problem, how to determine the place value of each numeral in a three-digit number, etc.</td></tr>
<tr><td colspan="2"><u>Formative Assessments</u>
Through what multiple sources of evidence will students demonstrate their understanding on a continual basis?
These help guide instruction and provide feedback to students.</td></tr>
<tr><td colspan="2">Examples of possible formative assessments include:
Personal Connections Journal - At regular intervals, provide students with time to write in their math journals. Encourage students to make personal connections to their learning. Students write about what they are learning and how it connects to their daily lives and to the world around them.
Exit Cards - Provide students with exit cards at the end of class. Students demonstrate their understanding of concepts/skills they practiced during math class by completing one or two questions related to the topic being studied.
Teach Someone - Ask students to “teach” a partner how to complete a question or practice a skill. Observe students to determine the depth of their understanding of the topic.</td></tr>
<tr><td colspan="2"><u>Pre-Assessments</u>
Pre-assessments are used to determine what students know and their readiness level to inform instruction.</td></tr>
<tr><td colspan="2">Examples of possible pre-assessments include:
Entry Cards - Before beginning the study of new topics or concepts, give students entry cards with questions related to the topics or concepts. Look through student responses to determine what the students know.
Write Something - Ask students to “write something” about a concept or skill that your students will be learning about in an upcoming topic or unit.
Say Something - For students who struggle with getting their ideas down on paper, ask them to “say something” about a concept or skill that they will be learning about.</td></tr>
</table>

STAGE THREE: CREATE THE LEARNING PLAN
Instructional Plan **The Instructional Plan should include a sequence of lessons, teaching strategies, and information on First Nation, Inuit and Metis Content integration and technology integration.**

The instructional plan will depend on the unique learning needs of the students in each Math 18 class. Through pre- and formative assessments, plan activities and learning opportunities that target your students' unique learning needs. Possible activities and strategies may include:

N18.1 Demonstrate an understanding of whole numbers to 1 000.
- Practice skip counting by 2's, 5's, 10's, and 100's.
- Use the Aces to 9s in a deck of playing cards. Students choose three of the cards and place them in a row. Students read the number shown by the cards.

N18.2 Represent, compare, and describe whole numbers to 1 000 within the contexts of place value and the base ten system.
- Give students different three digit number cards and have them represent the number using pictures and symbols.
- Play memory by matching three-digit numbers that have the same digit highlighted (hundreds, tens, or ones).

N18.3 Demonstrate an understanding of addition of whole numbers (up to 3 digits).
- Students draw a couple of three-digit number cards and add them together on a white board or scrap piece of paper.
- Have students write the equations for three numbers in a number family. For example, for the numbers 241, 319, and 560, students would write 241 + 319 = 560, 319 + 241 = 560, 560 - 319 = 241, and 560 - 241 = 319.

N18.4 Demonstrate an understanding of subtraction of whole numbers (up to 3 digits).
- Encourage students to practice estimating the difference between two numbers before solving the equation.
- Students write problems that involve subtraction and share them with a partner. The student who wrote the problems can correct them to ensure their partner's answer was right.

N18.5 Demonstrate an understanding of multiplication of whole numbers (up to 3 digits x 1 digit multipliers).
- Give students a multiplication equation and have them create an array to solve the equation.
- Encourage students to use a calculator to double-check their answers to the multiplication questions.

N18.6 Demonstrate an understanding of division of whole numbers (up to 3 digits divided by 1 digit divisors).
- Have students use concrete materials to represent a division equation. Encourage students to share their thinking about the strategy they used to solve the problem.
- Ask students to create a number of problems that involve division. Students can take turns solving the problems.

N18.7 Demonstrate an understanding of fractions less than or equal to one.
- Give students a number of cards with fractions less than or equal to one. Ask students to order the fractions from the smallest to the largest.
- Create a list of examples of fractions used in daily life. Refer to this list throughout the unit and have students add any new examples they come up with during the unit.

N18.8 Demonstrate an understanding of decimals to the hundredths.
- Provide students with opportunities to describe decimal numbers to the hundredths.
- Encourage students to change decimal numbers to fractions and fractions to decimal numbers. Ask students to write the decimals to the hundredths place.

Key Resources
Support Materials: Math Makes Sense Math Strategies that Multiply: The Best of Times by Greg Tang Children Are Mathematical Problem Solvers by Lynae Sakshaug, Melfriend Olson, and Judith Olson Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Schools Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8 It All Adds Up: Engaging 8 to 12 Year Olds in Math Investigations

Patterns and Relations

UbD Planning Document

Good Spirit School Division UbD Unit Plan		
Teacher:	**Subject: Math**	**Grade: Ten – Level 18**
Unit Title: Patterns and Relations		
Context (ELA only):	**Type of Unit (ELA only):**	
Time Frame:		
STAGE ONE: IDENTIFY THE DESIRED RESULTS		
Outcomes Addressed in the Unit **SK curriculum outcomes can be copied and pasted, focuses highlighted.**		
PR28.1 Demonstrate an understanding of patterns and relations in tables, charts, diagrams, and graphs.		
Big Ideas/Enduring Understandings **What do you want students to understand and be able to use several years from now?** **What are the BIG ideas?**	**Essential Questions** **Open-ended questions that stimulate thought and inquiry linked to the content of the enduring understandings.**	
- Patterns are found all around us. - The structure of a pattern can be represented in a variety of ways.	- What is a pattern? - Why do we use patterns? - What patterns are relevant in our lives? - How can patterns be represented?	

Knowledge and Skills (Students will know and do…)

What key knowledge and skills will students acquire as a result of this unit?
(These may be indicators from the curriculum)

Knowledge (Students will know…) What key knowledge will students acquire as a result of this unit?	Skills (Students will know how to…) What key skills will students acquire as a result of this unit?
- the definition of a pattern. - the steps to creating a pattern.	- find patterns on a multiplication chart. - determine a pattern rule. - describe patterns. - extend a pattern. - identify a sorting rule in a Venn diagram. - use a Venn diagram. - explain data on a Venn diagram. - compare similarities and differences of patterns. - read a one-to-one correspondence graph.

STAGE TWO: DESIGN ASSESSMENT EVIDENCE

Assessment Evidence

Summative Assessments/Performance Tasks

Assessments of what students know and can do aligned to the outcomes. They are a snapshot in time used for reporting and evaluating.

Outcomes/Objectives	Co-construct criteria and/or rubrics with students whenever possible. Examples of Summative Assessments/Performance Tasks include:
PR28.1	**Venn Diagram** - Provide students with information on the patterns and relations unit and have them create a Venn diagram using the information.
	Create a Pattern - Encourage students to use all the information they have learned about patterns and create their own pattern. Ask them to explain the pattern they have created, how they would extend the pattern, what they learned about patterns, etc.
	Patterns In Everyday Life Book - Students create a book that includes patterns that they see in their daily lives. A brief explanation or description should be included with each pattern to explain why the pattern was included in their book. Students can provide these explanations or descriptions verbally or in writing.

Formative Assessments

Through what multiple sources of evidence will students demonstrate their understanding on a continual basis?
These help guide instruction and provide feedback to students.

Examples of possible formative assessments include:
Personal Connections Journal - At regular intervals, provide students with time to write in their math journals. Encourage students to make personal connections to their learning. Students write about what they are learning and how it connects to their daily lives and to the world around them.
Draw It - Provide students with a set of criteria and have students draw a pattern that matches the criteria.
Scavenger Hunt - Give students a list of patterns to find in books, the classroom, in the school, or in the community. Have students search for the patterns.

Pre-Assessments

Pre-assessments are used to determine what students know and their readiness level to inform instruction.

Examples of possible pre-assessments include:
Graffiti Walls - Create a graffiti wall and ask students to write and draw what they know about patterns.
Quick Write - Provide students with a scrap piece of paper and give them five minutes to quickly write everything they know about patterns and where they find patterns in the world around them.
Turn and Talk - Before beginning a unit on patterns, encourage students to "turn and talk" to their neighbour about everything they know about patterns.

STAGE THREE: CREATE THE LEARNING PLAN

Instructional Plan

The Instructional Plan should include a sequence of lessons, teaching strategies, and information on First Nation, Inuit and Metis Content integration and technology integration.

The instructional plan will depend on the unique learning needs of the students in each Math 18 class. Through pre- and formative assessments, plan activities and learning opportunities that target your students' unique learning needs. Possible activities and strategies may include:

PR28.1 Demonstrate an understanding of patterns and relations in tables, charts, diagrams, and graphs.
- Provide students with copies of a multiplication chart. Encourage students to find and describe different patterns in the multiplication chart.
- Have students study a completed Venn Diagrams to discover characteristics of these types of diagrams. Provide empty diagrams and have students put information about a given comparison in the correct places.

Key Resources

Support Materials:
Math Makes Sense
Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Schools
Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8

Shape and Space

UbD Planning Document

Good Spirit School Division UbD Unit Plan		
Teacher:	**Subject: Math**	**Grade: Ten – Level 18**
Unit Title: Shape and Space		
Context (ELA only):	**Type of Unit (ELA only):**	
Time Frame:		
STAGE ONE: IDENTIFY THE DESIRED RESULTS		
Outcomes Addressed in the Unit **SK curriculum outcomes can be copied and pasted, focuses highlighted.**		
SS18.1 Demonstrate an understanding of time. SS18.2 Demonstrate an understanding of symmetry. SS18.3 Demonstrate an understanding of regular and irregular 2-D shapes. SS18.4 Demonstrate an understanding of 3-D objects. SS18.5 Demonstrate an understanding of mass (kg and g). SS18.6 Demonstrate an understanding of linear measurement (mm, cm, and m). SS18.7 Demonstrate an understanding of capacity (ml and l).		
Big Ideas/Enduring Understandings **What do you want students to understand and be able to use several years from now?** **What are the BIG ideas?**	**Essential Questions** **Open-ended questions that stimulate thought and inquiry linked to the content of the enduring understandings.**	
- Time is important in our daily lives. - Different cultures use different devices to keep time. - We use measurement in our daily lives. - There are 3-D shapes all around us. - Solids can be sorted by their attributes.	- What kinds of devices are used for time keeping in different cultures? - How are calendars useful? - Why is time important in my life? - What is mass? - When do we use measurement in our daily lives? - What is the difference between regular and irregular polygons? - How are 2-D and 3-D objects related?	

Knowledge and Skills (Students will know and do…)

What key knowledge and skills will students acquire as a result of this unit?
(These may be indicators from the curriculum)

Knowledge (Students will know…) **What key knowledge will students acquire as a result of this unit?**	**Skills (Students will know how to…)** **What key skills will students acquire as a result of this unit?**
- the names of the months. - the names of the days of the week. - that time can be measured by standard and non-standard units. - time can be expressed in different ways. - the relationship between kg and g. - what a scale is and what it is used for. - when it is appropriate to estimate and when a precise measurement is needed. - to use skip counting when measuring. - the definitions of meter, centimetre, perimeter, mass, capacity, etc. - what a 2-D shape is. - what a 3-D shape is. - the names and characteristics of 2-D and 3-D shapes and objects.	- order the days of the week and the months of the year. - describe relevant activities that involve the measurement of time. - identify the day of the week, the month of the year, and the year for a given date. - locate dates on a calendar and explain the strategy used. - create a personal calendar of events. - create and solve problems involving estimating and measuring mass and capacity. - use a scale to determine the mass of an object. - describe the relationship between kg and g. - measure length, width, and perimeter. - sketch a line segment for a given length. - measure regular and irregular polygons. - construct 2-D shapes with a given perimeter. - construct 3-D shapes with a given area. - know the appropriate unit to use when measuring. - sort 3-D objects according to their faces, edges, and vertices and explain the sorting rule.

STAGE TWO: DESIGN ASSESSMENT EVIDENCE

Assessment Evidence

Summative Assessments/Performance Tasks

Assessments of what students know and can do aligned to the outcomes. They are a snapshot in time used for reporting and evaluating.

Outcomes/Objectives	Co-construct criteria and/or rubrics with students whenever possible. Examples of Summative Assessments/Performance Tasks include:
SS18.1, SS18.2, SS18.3, SS18.4, SS18.5, SS18.6, SS18.7	**Concrete Models** - Students create concrete models of 2-D shapes and 3-D objects. Interview students to determine what knowledge they have learned.
	Dioramas - Students create a diorama of objects in their everyday lives that showcase 3-D objects found in the school, home, and community.

	Measurement Centres - Set up measurement centres that focus on area, perimeter, capacity, mass, etc. Students rotate through the centres and record their responses. Use the responses to determine what students have learned about measurement during the shapes and spaces unit.

Formative Assessments

Through what multiple sources of evidence will students demonstrate their understanding on a continual basis?
These help guide instruction and provide feedback to students.

Examples of possible formative assessments include:

Personal Connections Journal - At regular intervals, provide students with time to write in their math journals. Encourage students to make personal connections to their learning. Students write about what they are learning and how it connects to their daily lives and to the world around them.

What Am I? Riddles - Provide students with what am I? riddles about different shapes, objects, and/or measurements. Students try to solve the riddles. Once students are familiar with the process, ask them to compose their own riddles.

3,2,1 Exit Slips - At the end of class, ask students to write or tell three things they learned, two questions they have about the topic, and one thing they found interesting.

Pre-Assessments

Pre-assessments are used to determine what students know and their readiness level to inform instruction.

Examples of possible pre-assessments include:

Brainstorming - Give students a topic or a question and have them brainstorm everything they know about it (e.g., uses of linear measurement in daily life, list of 3-D objects, etc.). This activity can be completed individually, in small groups, or as a class.

Carousel Brainstorming - Set up target topics found throughout the shapes and spaces unit that you want to pre-assess students on before beginning your unit of study. Have students rotate around the classroom and record their responses to the different topics.

Placemats - Provide groups of four students with a placemat graphic organizer. Encourage students to write or print what they know about measurement in their section of the placement. The groups then discuss their responses. After the discussion, students write one or two key agreed upon ideas in the centre of the placemat.

STAGE THREE: CREATE THE LEARNING PLAN

Instructional Plan

The Instructional Plan should include a sequence of lessons, teaching strategies, and information on First Nation, Inuit and Metis Content integration and technology integration.

The instructional plan will depend on the unique learning needs of the students in each Math 18 class. Through pre- and formative assessments, plan activities and learning opportunities that target your students' unique learning needs. Possible activities and strategies may include:

SS18.1 Demonstrate an understanding of time.

- Bring in different time pieces so students can see a variety of ways that time can be represented. Include digital and analog clocks, stop watches, 24-hour clocks, etc.
- Brainstorm a list of activities that are directly related to time. Discuss each event and order the events according to their timeline.

SS18.2 Demonstrate an understanding of symmetry.

- Discuss the concept of symmetry and bring in a number of examples for students to look at and study. Have students indicate whether the examples are in fact symmetrical and ask them to identify the reasons for their response.
- Provide pictures of symmetrical shapes and objects on cards that have half the shape or half the object removed. Students fill in the missing part of the picture so it is symmetrical.

SS18.3 Demonstrate an understanding of regular and irregular 2-D shapes.

- Give students a number of pictures with 2-D shapes on them. Have students sort the cards into different categories. Ask students to share the categories they have chosen with the rest of the class.
- Brainstorm a list of 2-D shapes found in the classroom, home, and community.

SS18.4 Demonstrate an understanding of 3-D objects.

- Provide students with a number of 3-D objects and have them sort the objects based on specific attributes. See how many different ways the students can find to sort the objects.
- Brainstorm a list of 3-D objects found in the classroom, home, and community.

SS18.5 Demonstrate an understanding of mass (kg and g).

- Demonstrate the relationship between g and kg by bringing in a weigh scale and having students weigh different objects. Encourage students to share what they learn with their classmates.
- After students are familiar with g and kg, bring in objects or pictures of objects and have students identify the unit (g and kg) needed to weigh the object.

SS18.6 Demonstrate an understanding of linear measurement (mm, cm, and m).

- Provide students with a tape measure or meter stick. Ask students to measure the length of given objects. Encourage them to use the appropriate unit (mm, cm, and m) for each object.
- Measure the areas of triangles, rectangles, squares, and circles using given formulas.

SS18.7 Demonstrate an understanding of capacity (ml and l).

- Demonstrate the relationship between ml and l to the students. Have students identify when it is appropriate to use each unit of measure.
- Set up stations with different sized containers filled with different amounts of liquid. Have students use standard containers for measurement (graduated cylinders, measuring cups, measuring spoons, etc.) to determine the amount of liquid in each container. Encourage students to estimate the amount before the actual measurement.

Key Resources
Support Materials: Math Makes Sense Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Schools Geometry: Seeing, Doing, Understanding Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8

Statistics and Probability

UbD Planning Document

Good Spirit School Division UbD Unit Plan		
Teacher:	**Subject: Math**	**Grade: Ten – Level 18**
Unit Title: Statistics and Probability		
Context (ELA only):	**Type of Unit (ELA only):**	
Time Frame:		
STAGE ONE: IDENTIFY THE DESIRED RESULTS		
Outcomes Addressed in the Unit **SK curriculum outcomes can be copied and pasted, focuses highlighted.**		
SP18.1 Demonstrate an understanding of graphs. SP18.2 Demonstrate an understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots.		
Big Ideas/Enduring Understandings **What do you want students to understand and be able to use several years from now?** **What are the BIG ideas?**	**Essential Questions** **Open-ended questions that stimulate thought and inquiry linked to the content of the enduring understandings.**	
- We record and organize data in our daily lives. - Graphs make it easier to understand data. - Different cultures collect, represent, and use data in different ways. - Data can be collected, organized, and displayed in a variety of different ways.	- What are some ways that data can be represented? - What are some ways that different cultures collect, represent, and use data? - Why would we want to represent data on a graph?	

Knowledge and Skills (Students will know and do…)

What key knowledge and skills will students acquire as a result of this unit?
(These may be indicators from the curriculum)

Knowledge (Students will know…) What key knowledge will students acquire as a result of this unit?	Skills (Students will know how to…) What key skills will students acquire as a result of this unit?
- what a graph is. - the definitions of tally marks, charts, bar graphs, line plots, and pictographs.	- formulate questions that can be used to gather information. - read a graph. - construct a graph. - construct a questionnaire.

STAGE TWO: DESIGN ASSESSMENT EVIDENCE

Assessment Evidence

Summative Assessments/Performance Tasks

Assessments of what students know and can do aligned to the outcomes. They are a snapshot in time used for reporting and evaluating.

Outcomes/Objectives	Co-construct criteria and/or rubrics with students whenever possible. Examples of Summative Assessments/Performance Tasks include:
SP18.1, SP18.2	**Conduct a Survey** - Encourage students to come up with a question of interest to them and gather data by asking a number of people their survey question. Students then construct a graph and report on the process they went through to gather data and report on their findings.
	Data Analysis Project - Provide students with data and a graph and have them display the data and answer a series of questions related to the graph.
	Graphing Project - Students look through magazines and newspapers and on the internet for different types of graphs. Encourage students to identify the information that can be gained from reading the graph.

Formative Assessments

Through what multiple sources of evidence will students demonstrate their understanding on a continual basis?
These help guide instruction and provide feedback to students.

Examples of possible formative assessments include:

Personal Connections Journal - At regular intervals, provide students with time to write in their math journals. Encourage students to make personal connections to their learning. Students write about what they are learning and how it connects to their daily lives and to the world around them.

List 10 Things - Provide students with a graphic organizer and have them list ten things they know about graphs. Give students the list ten things graphic organizer on a number of occasions and compare the lists to determine what students are learning about graphs.

Fist of Five - At various points throughout the statistics and probability unit, encourage students to represent their understanding through the fist of five strategy.

Pre-Assessments

Pre-assessments are used to determine what students know and their readiness level to inform instruction.

Examples of possible pre-assessments include:

One Minute Essay - Students write for one minute on everything they know about graphs and their role in our home, school, and community.

KWL Chart - At the start of the statistics and probability unit, ask students to write what they know about graphs in the 'K' section of the KWL chart. Students can also list any questions or areas that they would like to learn about in the 'W' section of the chart. Revisit the chart throughout the unit and have students add additional information.

Email Questions - Before beginning a study of graphs, email a number of questions to students about graphing and have them answer the questions and email their responses back to you.

STAGE THREE: CREATE THE LEARNING PLAN

Instructional Plan

The Instructional Plan should include a sequence of lessons, teaching strategies, and information on First Nation, Inuit and Metis Content integration and technology integration.

The instructional plan will depend on the unique learning needs of the students in each Math 18 class. Through pre- and formative assessments, plan activities and learning opportunities that target your students' unique learning needs. Possible activities and strategies may include:

SP18.1 Demonstrate an understanding of graphs.

- As a class, establish a question that is relevant to the students. Conduct the survey and walk through the process of plotting the information on a graph.
- Have students work in partners or small groups to come up with a survey question, conduct the survey, and display the data gathered on a graph.

SP18.2 Demonstrate an understanding of first-hand data using tally marks, charts, lists, bar graphs, and line plots.

- Study a variety of graphs, tables, charts, and line plots. Look at the data used to help students gain an understanding of first-hand data.
- Provide students with data and have them place the data on bar graphs and line plots. Discuss what the data is telling those who study the graphs and line plots. Explain how there are different ways to interpret the data.

Key Resources

Support Materials:

Math Makes Sense

Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Schools

Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8

Consumer Math

UbD Planning Document

Good Spirit School Division UbD Unit Plan		
Teacher:	**Subject: Math**	**Grade: Ten – Level 18**
Unit Title: Consumer Math		
Context (ELA only):	**Type of Unit (ELA only):**	
Time Frame:		
STAGE ONE: IDENTIFY THE DESIRED RESULTS		
Outcomes Addressed in the Unit **SK curriculum outcomes can be copied and pasted, focuses highlighted.**		
CM18.1 Demonstrate an understanding of coins and bills up to $100.00. CM18.2 Apply understanding of money.		
Big Ideas/Enduring Understandings **What do you want students to understand and be able to use several years from now?** **What are the BIG ideas?**	**Essential Questions** **Open-ended questions that stimulate thought and inquiry linked to the content of the enduring understandings.**	
- Money plays an important role in our daily lives.	- How do you determine what coins or bills to use for a purchase?	
Knowledge and Skills (Students will know and do…) **What key knowledge and skills will students acquire as a result of this unit?** **(These may be indicators from the curriculum)**		
Knowledge (Students will know…) **What key knowledge will students acquire as a result of this unit?**	**Skills (Students will know how to…)** **What key skills will students acquire as a result of this unit?**	
- the names and values of coins and dollar bills up to $100.00. - the steps to problem solve.	- count a given amount of money. - make change for a given amount of money. - determine the cost of multiple items. - compare the price of items in different sizes and multiples. - solve problems that involve the use of money.	
STAGE TWO: DESIGN ASSESSMENT EVIDENCE		

Assessment Evidence

Summative Assessments/Performance Tasks

Assessments of what students know and can do aligned to the outcomes. They are a snapshot in time used for reporting and evaluating.

Outcomes/Objectives	Co-construct criteria and/or rubrics with students whenever possible. Examples of Summative Assessments/Performance Tasks include:
CM18.1, CM18.2	**Brochure** - Students create a brochure that demonstrates their understanding of the outcomes studied during the consumer math unit.
	Photo Journal - Students create a photo journal of concepts and skills related to money. They can take pictures related to the topic and then journal about their choices.
	Quiz - Provide students with a list of problems involving money. Students use their understanding of consumer math to solve the problems.

Formative Assessments

Through what multiple sources of evidence will students demonstrate their understanding on a continual basis?
These help guide instruction and provide feedback to students.

Examples of possible formative assessments include:

Personal Connections Journal - At regular intervals, provide students with time to write in their math journals. Encourage students to make personal connections to their learning. Students write about what they are learning and how it connects to their daily lives and to the world around them.

Grocery List - Provide students with copies of flyers. Give students a set amount of money to work with and have them plan their meals for a week and price out cost of ingredients. Look at items on sale. Look at the different sizes of items. Which item is the better value? Point out that the bigger sizes are not always more cost-effective. Cost-effectiveness may depend on the expiration date of the item.

Role Play - Set up a store in the back of your classroom. Have students take turn running the store while other students make purchases. Storekeepers need to calculate the total cost of the items they sell while customers need to ensure they have enough money to buy the items they want to purchase.

Pre-Assessments

Pre-assessments are used to determine what students know and their readiness level to inform instruction.

Examples of possible pre-assessments include:

Word Sort - Provide students with a number of word cards that are related to the consumer math unit. Ask students to put the words into different categories. Students share their categories with a partner, in small groups, and/or with the entire class.

Anticipation Guide - Give each student an anticipation guide and have them determine if each statement is “true” or “false.” If statements are “false,” ask students to rewrite the statements so they are “true.”

One Word Web - Students complete a one word web on the topic of money. They think of all the words they can related to money and print them in the various ovals on the graphic organizer.

STAGE THREE: CREATE THE LEARNING PLAN
Instructional Plan **The Instructional Plan should include a sequence of lessons, teaching strategies, and information on First Nation, Inuit and Metis Content integration and technology integration.**
The instructional plan will depend on the unique learning needs of the students in each Math 18 class. Through pre- and formative assessments, plan activities and learning opportunities that target your students' unique learning needs. Possible activities and strategies may include: **CM18.1 Demonstrate an understanding of coins and bills up to $100.00.** - Ensure students understand the names of the coins and bills and their amounts by having them match up the coins and bills to the amounts. - Bring in a number of products and label them with their cost. Have students count out the appropriate amount of money for each item. **CM18.2 Apply understanding of money.** - Have students apply their understanding of money by solving a variety of problems that involve the cost of individual and multiple items. - Students compare the prices of similar products or different sizes of the same product.
Key Resources
Support Materials: Life Skills, Grades 5-8: Preparing Students for the Future Math Makes Sense Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Schools Good Questions for Math Teaching: Why Ask Them and What to Ask Grades 5-8 Learning Math With Calculators

Glossary

(taken from the Saskatchewan Online Math Curriculums)

Addend
Any quantity being added to another quantity (e.g., in the expression 32 + 57, both 32 and 57 are addends).

Angle
When two lines meet each other, angles are formed. The size of the angle is the amount of turn needed to take one line and place it on top of the other line. Angles can be classified by their measurement in degrees. Angles less than 90° are called acute angles.

Angles that measure 90° are called right angles. Angles that measure between 90° and 180° are called obtuse angles. Angles that measure 180° are called straight angles.

Array
A visual, concrete, or pictorial pattern arranged in a grid formation. For example, seating in an auditorium could be represented by an array.

Attributes
Characteristics of 2-D shapes and 3-D objects that can be used to compare and sort sets of 2-D shapes and 3-D objects (e.g., colour, relative size, number of corners, number of lines of symmetry).

Bar Graph
A graph in which data are represented by horizontal or vertical bars. Each bar represents the quantity of data of a certain type or category (e.g., red, blue, green, or yellow; 1998, 1999, 2000, 2001; or car, truck, bus, bike, walk).

Benchmarks
Numeric quantities used to compare and order other numeric quantities. For example, 0, 5, 10, and 20 are often used as benchmarks when placing whole numbers on a number line.

Chart
"Chart" is a general word for various kinds of pictures and diagrams which are used to represent data - often frequency data (e.g., a bar graph can be referred to as a bar chart).

Correspondence
A correspondence is a description of how one set of numbers (or objects) is mapped to a second set of objects. For example, a correspondence might describe how individual students are matched to their shoes. If each child in the class has a different type of shoe, then the correspondence between the shoes and the students would be one-to-one (for every child, there is exactly one type of shoe). If some children have the same type of shoe, then the correspondence is said to be many-to-one (many children to one type of shoe).

Denominator

The bottom number in a fraction that defines how many equal parts are in a whole.

Dividend
In a division statement, the dividend is the quantity that is being divided into equal groups. For example, in the expression 38 ÷ 4, 38 is the dividend (which is being divided into groups of 4 items or being divided into 4 equal groups).

Divisor
In a division statement, the divisor is the number of groups to be created, or the number of items to be within a group. For example, in the expression 38 ÷ 4, 4 is the divisor.

Double Bar Graph
A bar graph which compares two different sets of data according to the same criteria. For example, in a double bar graph, internet usage of boys and girls at a particular school could be compared over a number of years (the criteria).

Equality as a Balance and Inequality as Imbalance
The equal sign represents the idea of equivalence. For many students, it means "do the question". For some students, the equal sign in an expression such as 2 + 5 = means to add. When exploring equality and inequality, by using objects on a balance scale, students discover the relationships between and among the mass of the objects. The equal sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true. When the scale is imbalanced, the equation is not true. Using 2 + 5 = □, rather than simply 2 + 5 = helps students understand that the equal sign (=) represents equality rather than "do the work" or "do the question".

Equation
An equation is a statement which says that one expression or quantity is equal to another.

First-hand Data
Data that has been directly collected by the person using it (e.g., questionnaire data).

Fraction
A description of quantity as part of a whole.

Interdisciplinary

Disciplines connected by common concepts and skills embedded in disciplinary outcomes.

Line Graphs
A graph that results from connecting consecutive ordered pairs in a relation using straight lines.

Line Plot
A stylized version of a pictograph where the symbol * is used in place of one picture.

Line Symmetry
A property of some 2-D shapes in which a line can be used to divide the 2-D shape in half. Although there are many types of symmetry, in grade four the students are focussing on symmetry that can be defined as a reflection through the line of symmetry (i.e, no rotation or translation involved).

Minuend
In a subtraction sentence, the quantity that is being decreased (e.g., in the subtraction sentence 84 – 55, 84 is the minuend).

Multidisciplinary
Discipline outcomes organized around a theme and learned through the structure of the disciplines.

Multiple
A multiple of a number is exactly divisible by that number with no remainder.

Multiplier
Any quantity that another quantity is being multiplied by.

Number, Numeral, Digit
A number is the name that we give to quantities. For example, there are seven days in a week, or I have three brothers – both seven and three are numbers in these situations because they are defining a quantity. The symbolic representation of a number, such as 287, is called the numeral. If 287 is not being used to define a quantity, we call it a numeral.

Numerals, as the symbolic representation of numbers, are made up of a series of digits. The Hindu- Arabic number system that we use has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. (Note: sometimes students are confused between these digits and their finger digits –

this is because they count their fingers starting at one and get to ten rather than zero to nine.) These digits are also numerals and can be numbers (representing a quantity), but all numbers and all numerals are combinations of digits. The placement of a digit in a number or numeral affects the place value of the digit and, hence, how much of the quantity that it represents. For example, in 326, the 2 is contributing 20 to the total, while in 236 the 2 contributes 200 to the total quantity.

Numerator
The top number in a fraction. The numerator tells how many (the quantity) parts are present or being considered. The number of parts making up the whole is defined by the denominator of the fractions.

Object
Object is used to refer to a three-dimensional geometrical figure. To distinguish this meaning from that of shape, the word "object" is preceded by the descriptor "3-D".

Pattern Rule
A description of how consecutive terms or elements in a pattern are determined.

Percent
Percent means out of a hundred, being part of a hundred. The symbol for percent is %.

Personal Strategies
Personal strategies are strategies that the students have constructed and understand. Outcomes and indicators that specify the use of personal strategies convey the message that there is not a single procedure that is correct. Students should be encouraged to explore, share, and make decisions about what strategies to use in different contexts. Development of personal strategies is an indicator of the attainment of deeper understanding.

Pictograph
A graph which uses pictures or symbols to show how often something occurs.

Polygon
2-D shapes that have straight line edges that only intersect at the endpoints of the line segments and that form a closed shape.

Problem
A situation or context in which a solution strategy is not immediately known, but requires being sought after.

Quadrilaterals
Four-sided polygons.

Referents
A concrete representation of a unit of measure.

Regular/Irregular 2-D Shapes
Regular 2-D shapes are those shapes whose side lengths are equal and angle measures are also equal. Irregular 2-D shapes do not have all equal side lengths and/or all equal angle measures. For example, a square is a regular 2-D shape, while not all rectangles are regular 2-D shapes.

Relation
A statement that explains how the terms in a pattern relate to each other or how one unknown value is related to another.

Representations
Mathematical ideas can be represented and manipulated in a variety of forms including concrete manipulatives, visual designs, sounds and speech, physical movements, and symbolic notations (such as numerals and operation signs). Students need to have experiences in working with many different types of representations, and in transferring and translating knowledge between the different forms of representations.

Subtrahend
In a subtraction statement, the quantity that is being subtracted (e.g., in the subtraction statement 90 – 26, 26 is the subtrahend).

Superimposing
This term refers to the covering of one 2-D shape by one or more other 2-D shapes by placing the new shapes on top of the original.

Table

A set of data specially laid out in rows and columns, so that given one value, another separate value can be read out which is connected to the first one in some way (e. g., multiplication table).

Tally Marks
One way to collect and organize data. Each tally mark (often shown as a downward stroke |) represents one time that value appears in the data . Frequently, tally marks are grouped into sets of fives (four downward strokes and one cross-stroke) for ease of counting.

Trans-disciplinary
All knowledge interconnected and interdependent; real-life contexts emphasized and investigated through student questions.

References

Grade Three Saskatchewan Online Curriculum

Grade Four Saskatchewan Online Curriculum

Grade Five Saskatchewan Online Curriculum

Grade Six Saskatchewan Online Curriculum

Grade Seven Saskatchewan Online Curriculum

Grade Eight Saskatchewan Online Curriculum

Grade Nine Saskatchewan Online Curriculum